LAPTAG

Lecture notes on Waves/Spectra Noise, Correlations and ….

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Lecture 5, March 27, 2004

The spectra can tell us if there is a signal present in the data and what the frequencies of the waves which are in the noise. Are there other techniques we can use that tell us more about the waves in the noise?

1) How long do they last before they disappear?

   Answer   Autocorrelation functions

2) Can we separate one wave from the noise and look at it?

   Answer   Digital Filtering and Cross Spectral Functions

3) Can we find the dispersion relation (how the frequencies and wavelengths are related) from studying the noise?

   Answer   Two point Correlation Functions.
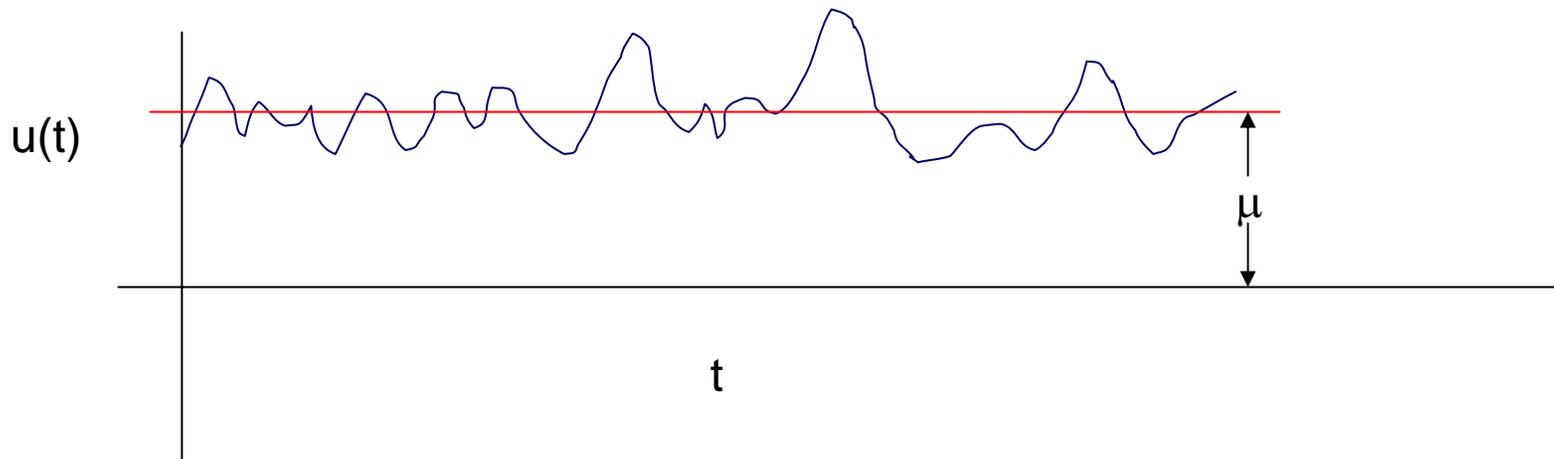
Let us examine these techniques one by one

Since we are interested in waves and they are all sines and cosines, the first thing we must do is subtract out the average value of our data from the data record.
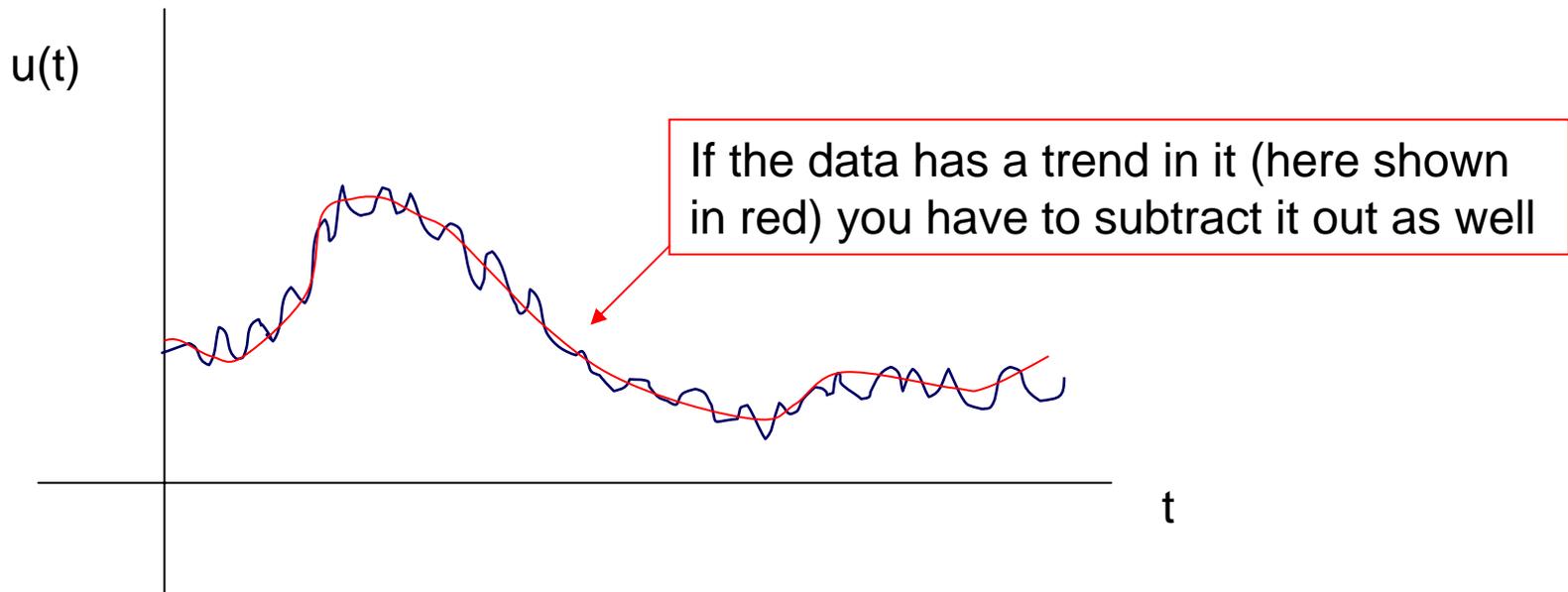
If there are n time steps then:

$u\left(t_{n}\right)$ is the data we just digitized

$$\langle u \rangle \equiv \mu = \frac{1}{N} \sum_{n=1}^{n} u\left(t_{n}\right) \quad \text{average}$$

$$\mathrm{x}\left(\mathrm{t}_{\mathrm{n}}\right) = u\left(t_{n}\right) - \mu \quad : \text{data with mean value removed}$$

u(t)

t

μ

u(t)

If the data has a trend in it (here shown in red) you have to subtract it out as well

t

There are procedures for this but this wont happen in our experiment so we will not discuss how to do it

The autocorrelation function tells us the time interval over which a correlation in the noise exists. If the noise is made entirely of waves, and the waves move through the plasma (or other medium) without decaying as they travel, the autocorrelation will be large for all time.

For digital data the autocorrelation is defined as:

You get one value of R for each delay time τ

$$R_{xx}(\tau) = \lim_{N \to \infty} \frac{1}{N} \sum_{t=1}^{N} x(t)x(t+\tau) \quad \text{If x is continuous}$$

IF x is digital and the data is taken at intervals $r\Delta t$

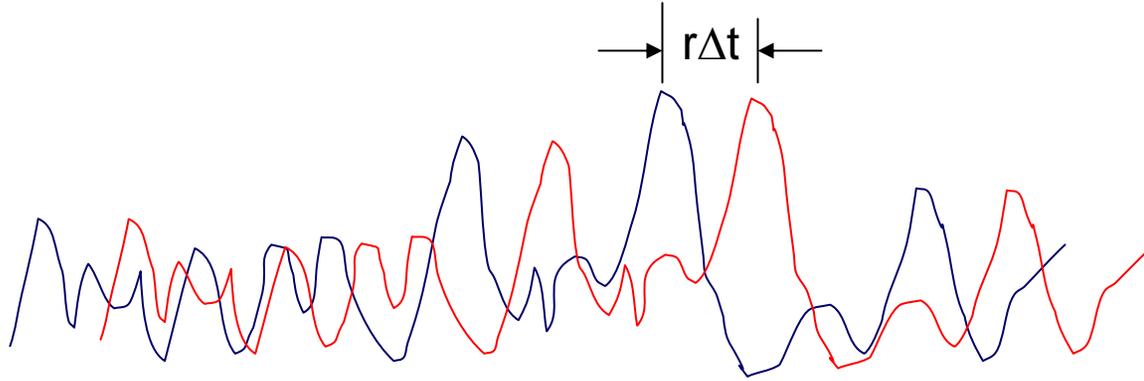(suppose Δt is .01 second then $r\Delta t=0,.01,.02,.03$ when $r =0,1,2,3..$)

$then:$

$$R_{xx}(r\Delta t) = \frac{1}{N-r} \sum_{n=1}^{N-r} x(t)x(t+r\Delta t) \equiv \frac{1}{N-r} \sum_{n=1}^{N-r} x_n x_{n+1}$$

r= 0,1,2,3,…m   where m<n

What does this mean?

For a particular r∆t multiply the function x(r∆t) by the function x(t+r∆t)

Then do it again and again. For each term you have to slide the data over by a timestep of ∆t and multiply all the terms. Note that all the term to the right of the "red" data are zero as are the terms to the left of the blue data. For the autocorrelation function there is only time history of data. We do this to find out how long the data are correlated for.
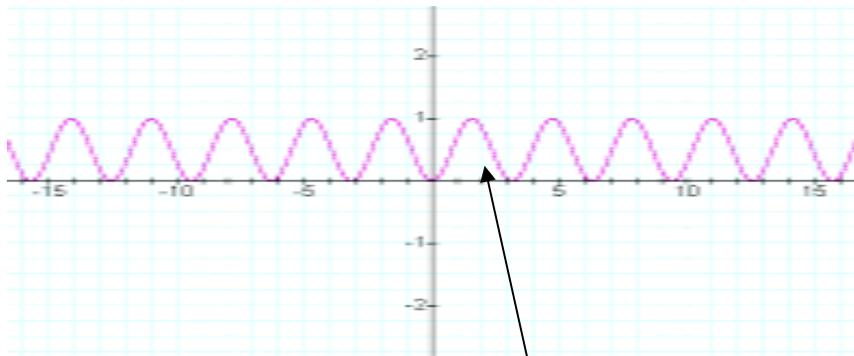
Note the $1/(N-r)$ guarantees that when $r=0$ R_{xx} is 1.0 !

What is the correlation of a sine wave with itself?

Let us assume that the sine wave is continuous.
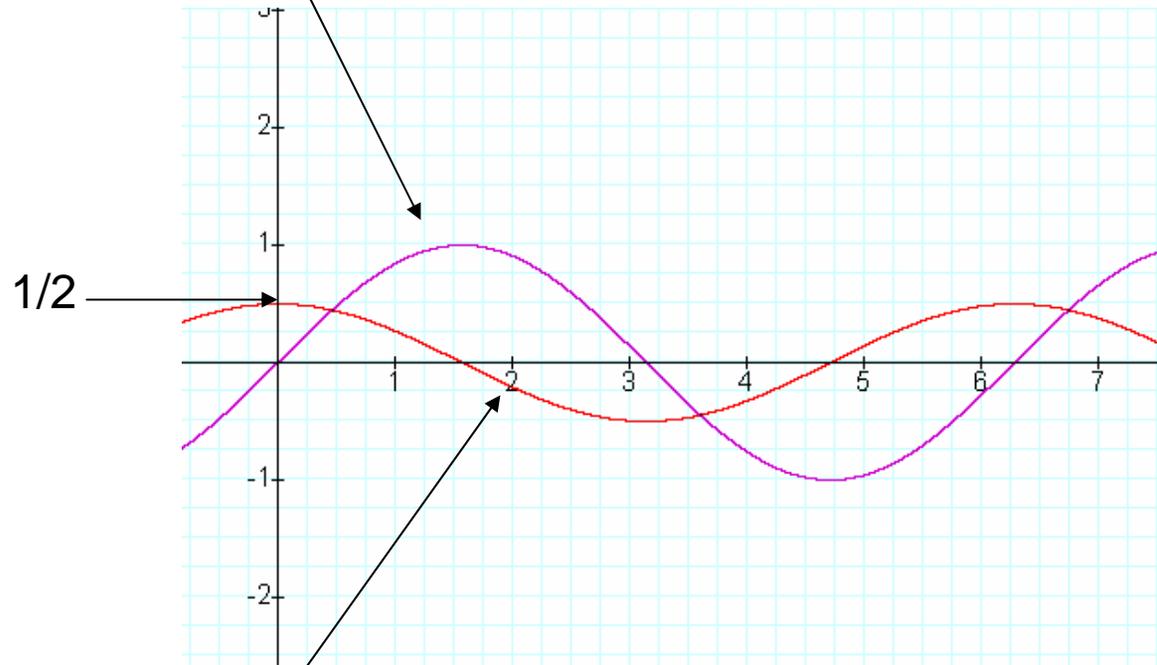
When r=0 (first point) we get $R_{xx}(0) = 1/2$ why?

All the terms go like $sin(t)*sin(t)=sin^2(t)$

The area under each lump is 1/4

There are 2 lumps per-period so the area under the period is ½. If there are N periods we have to divide the outcome by N. Therefore $R_{xx}(0) = 1/2$

$y = \sin(2\pi t/T)$ Original function (note it goes on forever!)

1/2

$R_{xx} = 1/2\sin(2\pi t/T)$ Autocorrelation function-calculated

From the definition (Try it can you get this?)

Now lets look at the digital sine wave data we used as an example for the FFT.

What will the autocorrelation look like?   Can you guess?

To do the correlation in PVwave is very easy. If the sine wave is called ysin and has 512 data points then all you have to do in wave is type:

ycor=autocorrelation(ysin,511)

This creates an array which is 512 points long and is the autocorrelation of ysin (the first argument).

Red=sine wave data

White:

autocorrelation
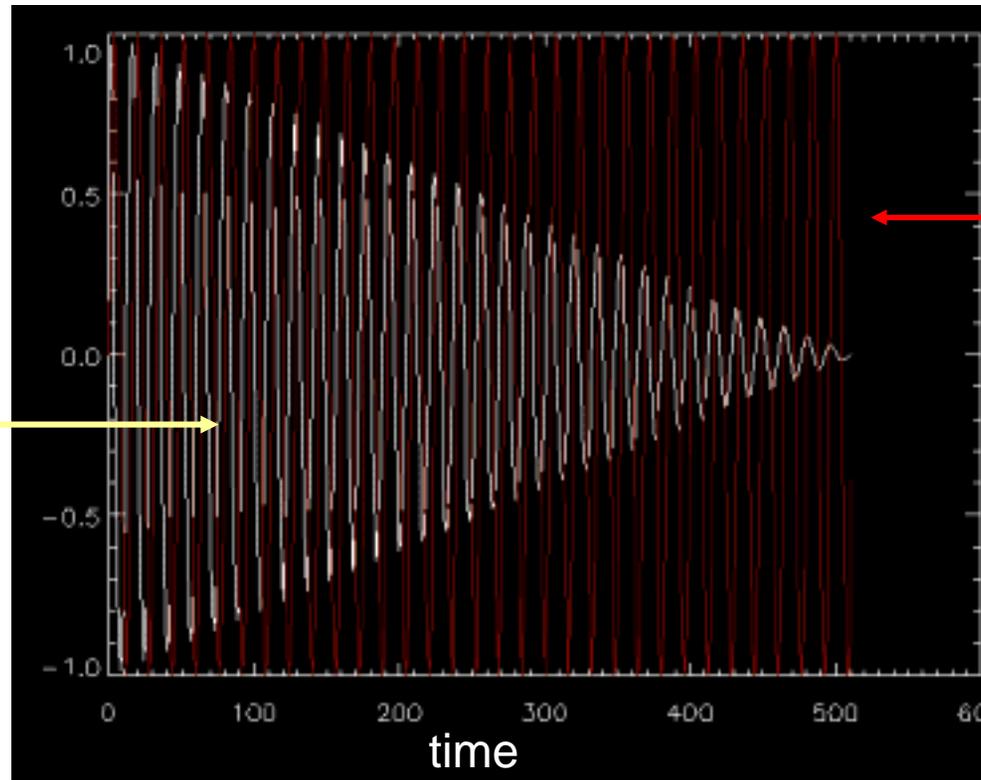
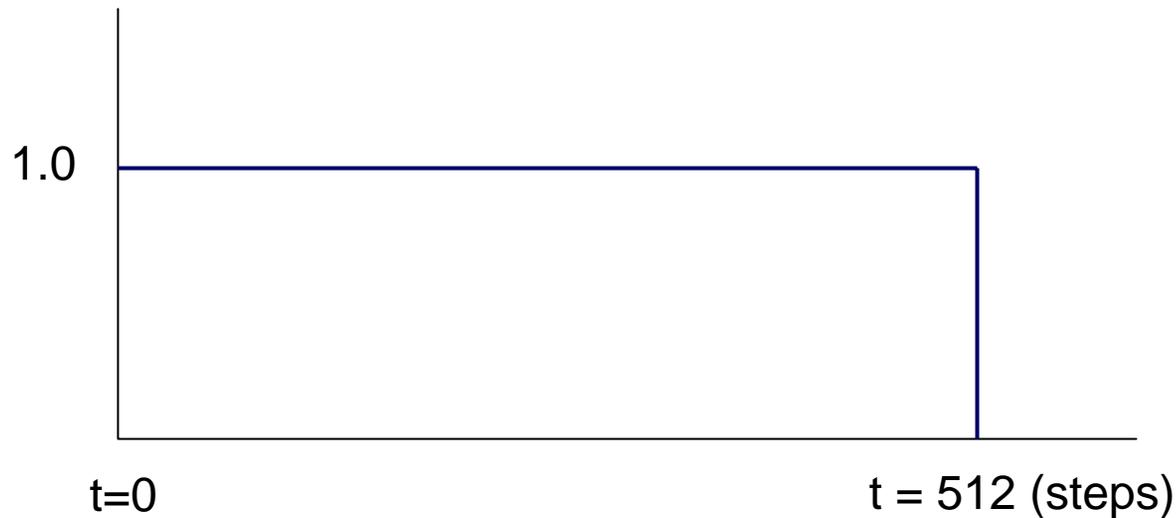time

Why does the autocorrelation decay in time ?

Answer: because the data does not go on forever!

What we have is a sine wave multiplied by a rectangular function which is 1 from t>0 until the 512 points and then it is zero!
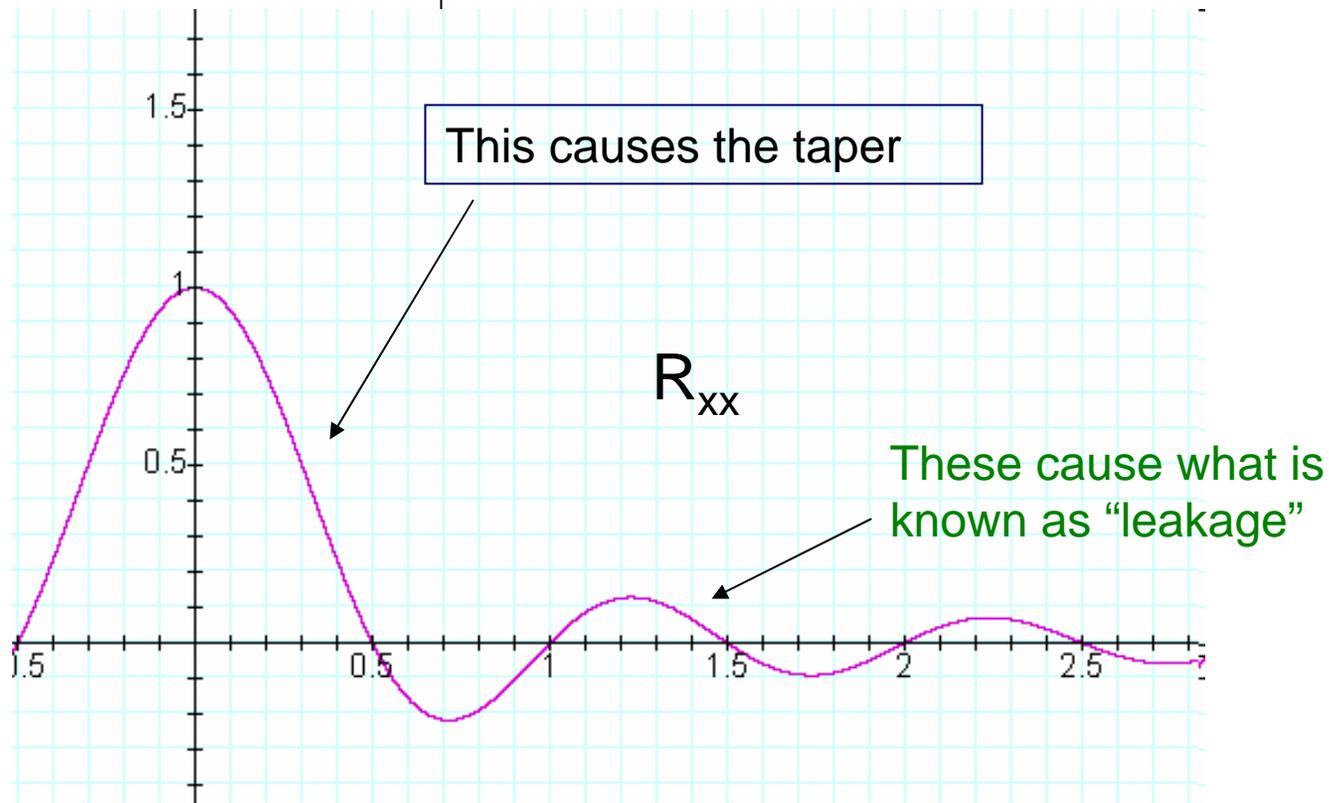

1.0

t=0   t = 512 (steps)

Autocorrelation of:

Random "white noise" only with a time interval x

$$y = \frac{\sin(2\pi x)}{(2\pi x)}$$

This causes the taper

R_{xx}

These cause what is known as "leakage"
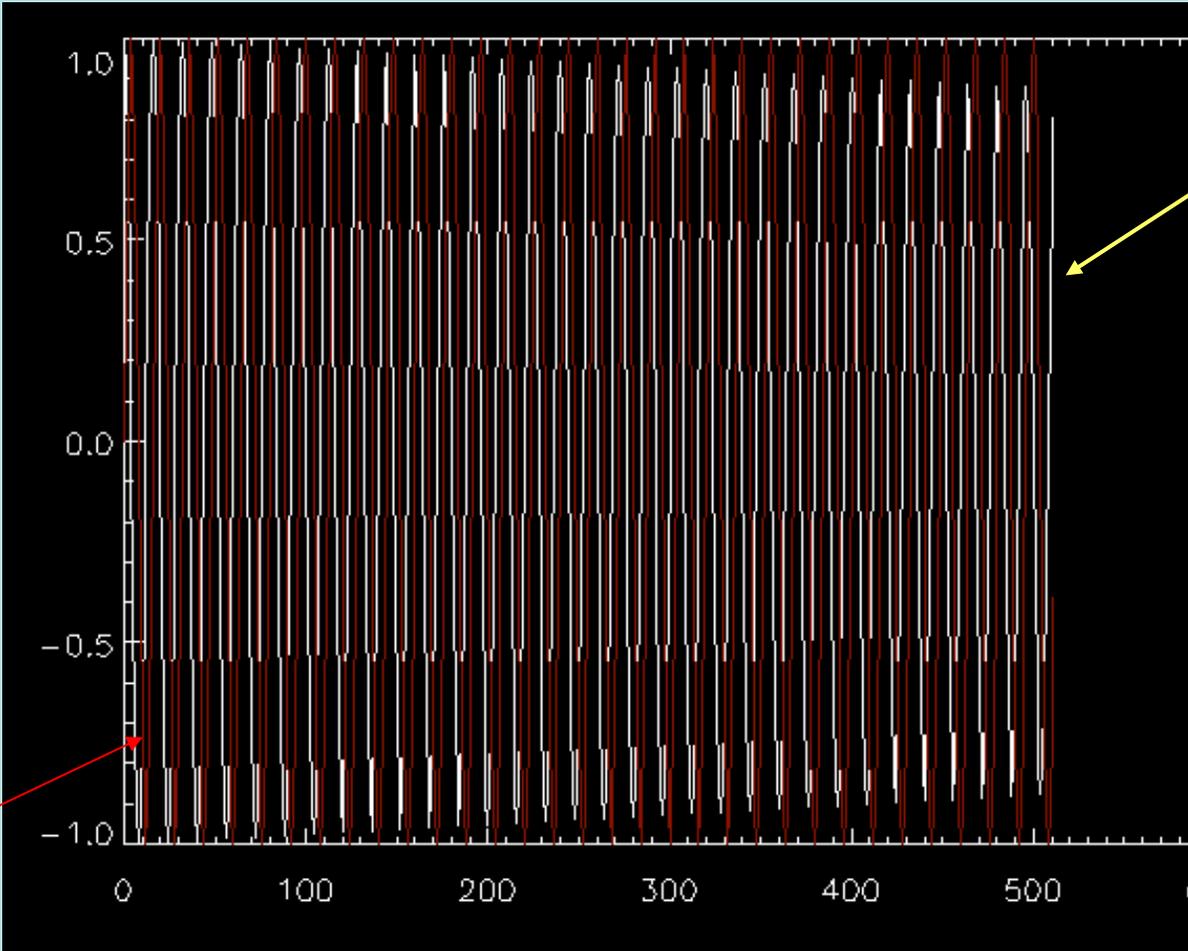
Therefore the cosine function of the infinitely long (in time) autocorrelation of the sine wave is multiplied by this, which tapers it off.

How do we get around this? Take a the longest possible record. This widens the first lobe of the sine wave. This is the same sinx and autocorrelation when the record length is 4096.

Autocorrelation

Of first 512

points

Orig.

Sine
wave

This is the autocorrelation of the magnetic field shown previously (and in red).
Note here the record is 5000 time steps long. It is correlated for less than 50 time
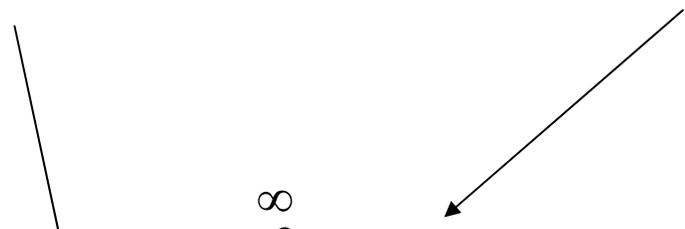steps (1/100 of the time)
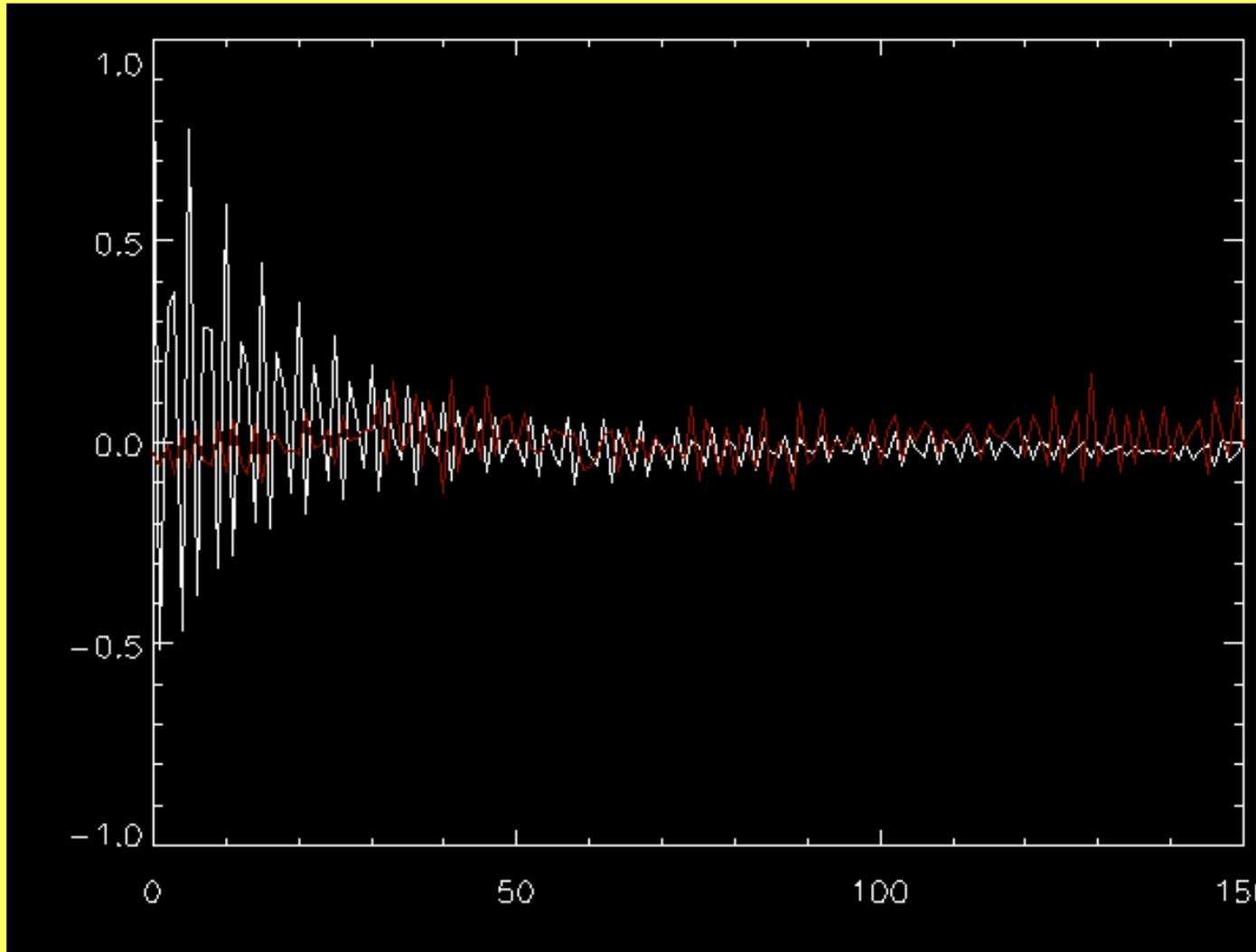
Amazing Fact !

The autocorrelation function and spectra are related to each other by

The   Fourier Transform!!

Spectra of x(t)x(t+rΔt) = Fourier Transform of the Autocorrelation function

Formally

$$S_{xx}(f) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-i2\pi f \tau} d\tau$$

This was proven by Von Neuman and is part of the mathematics of cybernetics or Information. This was essential for the development of computers