Quantum Statistical Mechanics for Stellar Astrophysics

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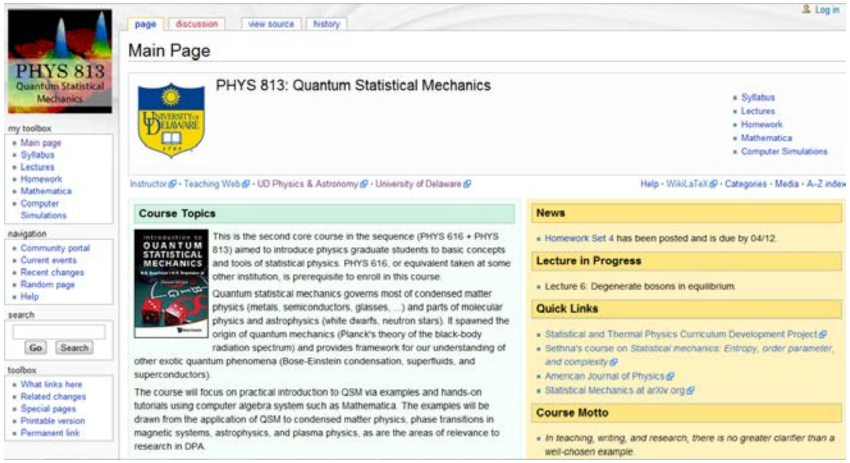
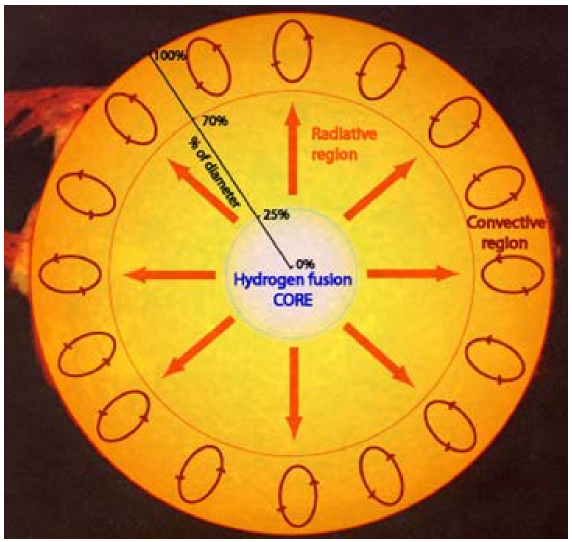
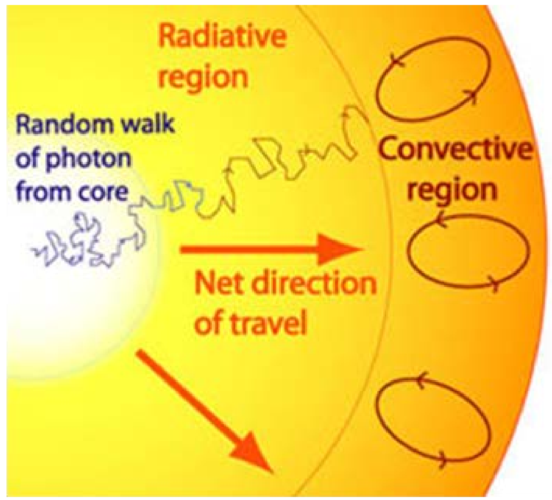


Light Emitted by Starts: Why is the Radiation from the Sun so Stable?

❑STELLAR ASTROPHYSICS: four fundamental forces of nature come into play in a characteristic and spectacular manner → the stars are formed by a collapse of matter caused by gravitational attraction; the light that they emit is generated by electromagnetic interactions; strong interactions provide their main source of energy; and weak interactions contribute in a crucial way to make their lifetime so long.



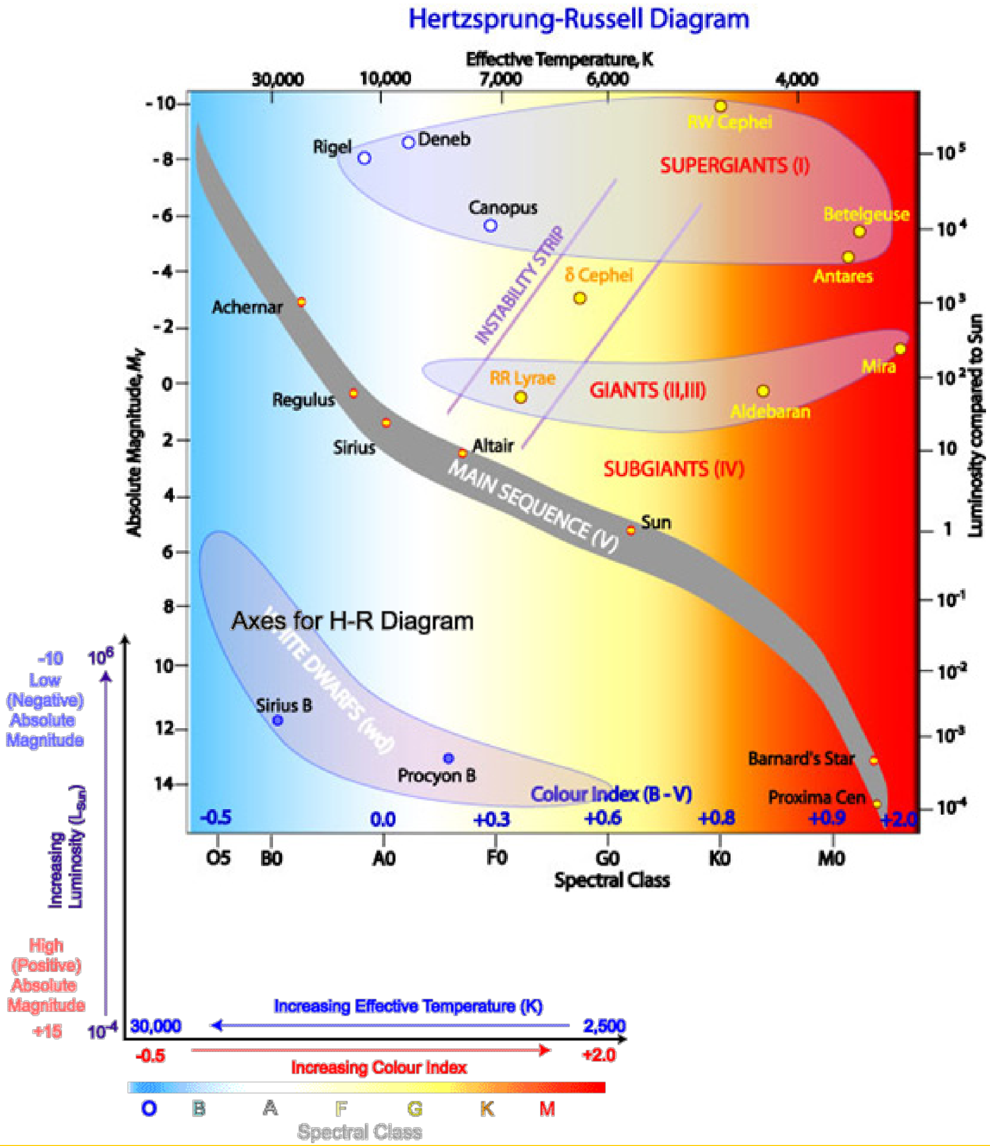


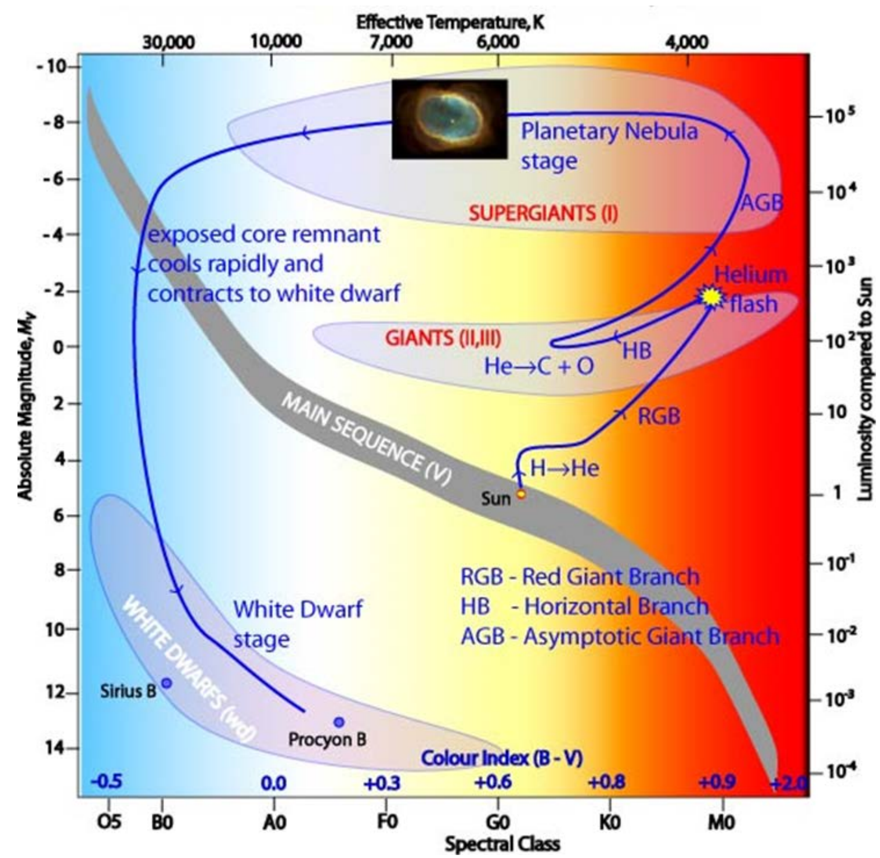
Sun as a main sequence star:

It is useful to keep in mind various orders of magnitude corresponding to the three principal classes of stars. Our Sun is a typical example of the so-called *main sequence*. Its radius is $R_\odot = 7\times10^8$ m, its mass is $M_\odot = 2\times10^{30}$ kg, its luminosity is $L_\odot = 3.8\times10^{26}$ W, and its surface temperature is $T_{s,\odot} = 6000$ K. Hence, its average density is $1.4\,\mathrm{g\,cm^{-3}}$, comparable to that of water on earth. The masses of all the stars lie between $0.1\,M_\odot$ and $100\,M_\odot$. The Sun is mainly made of hydrogen, with 28% of its mass consisting of ^{4}He nuclei and 2% of other light elements. Its number of protons is the order of 10^{57}. Heavier stars include red giants, a branch detached from the main sequence.

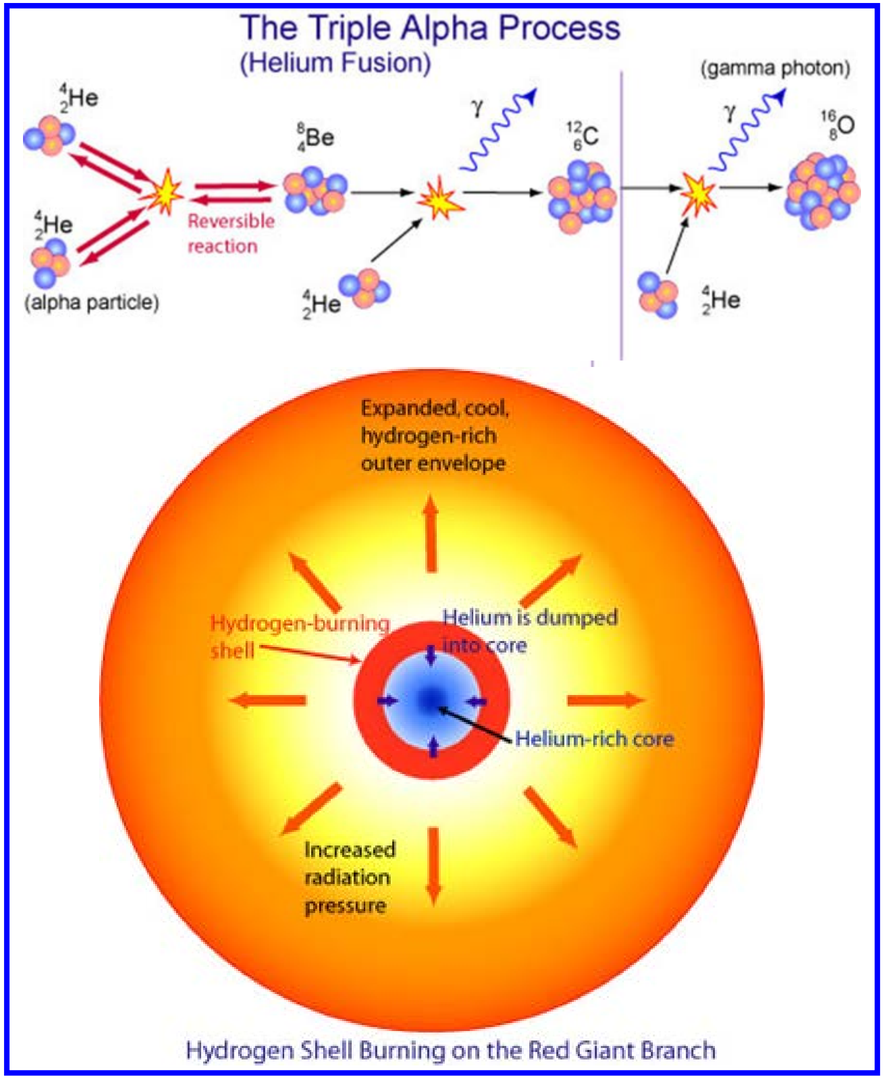
❑**1937 by Bethe and von Weiszäcker:** nuclear fusion of hydrogen into helium that take place in the central part of the Sun produce some amount of heat per unit time, which is exactly equal to the luminosity because the state of the Sun is stationary. However, such reactions are very sensitive to small changes in the temperature: they are activated by a rise, hindered by a decrease. **Thus, if it happens at some instant that a little more power is produced in the core than what is evacuated by radiation from the surface, why does the internal temperature not rise, eventually resulting in an explosion of the Sun? Conversely if the opposite perturbation occurs, why does the Sun not become extinct?**

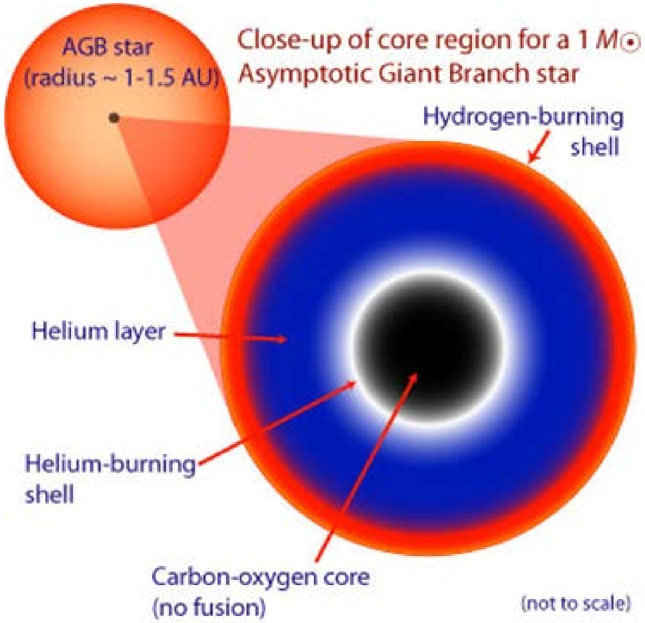
Hertzsprung-Russell Diagram and Evolution of Solar-Mass Stars

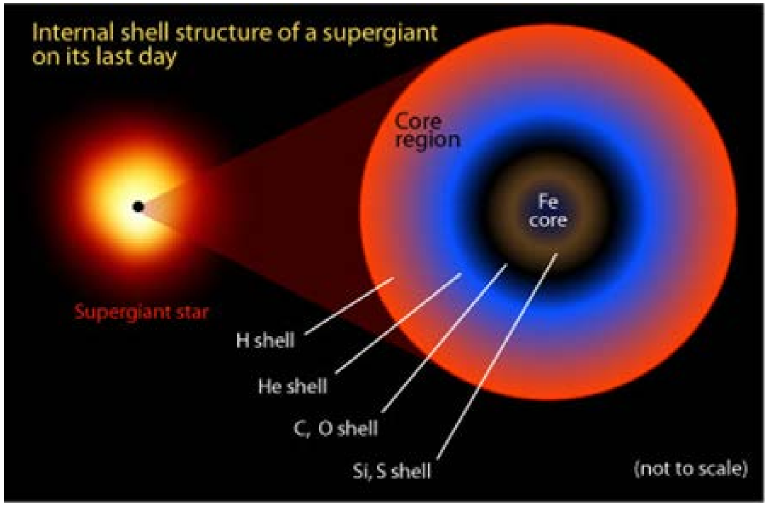




Internal Structure of Post-Main Sequence Stars in Pictures







Classical Pressure Acting Against Gravity in Main Sequence Stars

$$U \approx -\frac{GM^2}{R}$$

star wants to contract to a state of lower energy (i.e., larger negative values of U), unless there is outward direct pressure to resist the contraction

$$PV = Nk_BT$$

in ordinary stars with fuel for thermnuclear fusion, outward pressure is provided bythermal motion

$$\frac{\rho GM}{2R} \approx \frac{Nmk_BT}{Vm} = \frac{\rho k_BT}{m}$$

gravitation pressure at the center of the star is equal to thermal pressure

$$\frac{GM^2}{R} \approx -U \approx Nk_BT$$

total thermal energy is comparable to the magnitude of total gravitational potential energy

Insufficient Pressure → Black Holes and Their Thermodynamics

$$\left(\varepsilon - G\frac{\varepsilon/c^2\,M}{R_0}\right) \le 0$$

photon of energy ε cannot escape the surface of the star to reach an observer c

$R_0 \approx 3\text{ km for } M = M_\odot$

$$R_0 = \frac{2GM}{c^2}$$

Schwarzschild radius

factor 2 comes from general relativity which must be used when gravitation potential energy is comparable to other energies

$$S = \frac{k_B c^3}{4G\hbar}A$$

Bekenstein-Hawking entropy of a black hole

$$\frac{1}{T} = \frac{\partial S}{\partial E} = \frac{1}{c^2}\frac{\partial}{\partial M}\left[\frac{k_B c^3}{4G\hbar}4\pi\left(\frac{2GM}{c^2}\right)\right] = \frac{8\pi k_B G}{\hbar c^3}M$$

$$T = \frac{M_\odot}{M}6.169\times10^{-8}\text{ K}$$

HAWKING RADIATION: Particle-antiparticle pairs created from vacuum normally quickly annihilate each other, but near the horizon of a black hole, it's possible for one to fall in before the annihilation can happen, in which case the other one escapes as Hawking radiation

$$\frac{1}{A}\frac{\partial E}{\partial t} = -\sigma T^4 \Rightarrow \frac{d}{dt}\left(Mc^2\right) = -\frac{\pi^2 k_B^4}{60\hbar^3 c^2}\left(\frac{16\pi G^2}{c^4}\right)\left(\frac{\hbar c^3}{8\pi k_B G}\frac{1}{M}\right)^4$$

Hawking radiation evaporates black hole

$$M^2\frac{dM}{dt} = -\frac{\hbar c^4}{15360G^2} \equiv -b \Rightarrow M(t) = (M_\odot^3 - 3bt)^{1/3} \to 0 \text{ after } \tau \approx 2.2\times10^{74}\,s$$

much longer than the age of the Universe ~10^{18}

White Dwarfs Stabilized by Fermi Pressure of Non-Relativistic Electrons

$A, Z, A-Z, x = Z/A$ atomic number, number of protons, number of neutrons, electron fraction

$M \approx Nm_p, N_e = xN$ mass of star, number of electrons

$$\varepsilon_F = \frac{p_F^2}{2m_e} = \frac{h^2}{2m_e}\left(\frac{3N_e}{8\pi V}\right)^{2/3}$$ Fermi energy of electron gas in non-relativistic approximation

$$E_e = \frac{3}{5}N_e\varepsilon_F$$ Total kinetic energy of electron gas

$$E(R) = \frac{3}{5}N_e\varepsilon_F - \frac{3GN^2m_p^2}{5R}$$ Total energy of cool star where thermal energy can be neglected

$$\frac{d}{dR}E(R) = 0 \Rightarrow R = \frac{xh^2}{4m_e}\left(\frac{9}{4\pi^2}xN\right)^{2/3}\frac{1}{GNM_p^2}$$ White dwarfs cool off and contract to this radius

$$M = 0.85M_\odot, N = 10^{57}, x = \frac{1}{2} \Rightarrow R \approx 8000 \text{ km}, \rho = 3\times10^6 \text{ g/cm}^3$$

White Dwarfs Stabilized by Fermi Pressure of Relativistic Electrons

$$p_F = h\left(\frac{3N}{8\pi V}\right)^{1/3}$$

$$\varepsilon_F = \sqrt{c^2 p_F^2 + m_e^2 c^4} - m_e c^2 \xrightarrow{p_F \gg m_e c} p_F c$$

energy per electron increases with N, so when it becomes comparable to mc² we have to switch to relativistic energy-momentum dispersion

$$E(R) = \frac{3}{8}\frac{xNhc}{R}\left(\frac{9}{4\pi^2}xN\right)^{1/3} - \frac{3GN^2 m_p^2}{5R}$$

for simplicity we assume uniform density assumed, while in reality density is larger in the center of the star than further out

both terms depend on R, so total anergy of star decreases continously with decreasing radius

$$E(R) = 0 \Leftrightarrow \frac{3}{8}\frac{xN_c hc}{R}\left(\frac{9}{4\pi^2}xN_c\right)^{1/3} = \frac{3GN_c^2 m_p^2}{5R} \Rightarrow N_c = \frac{3}{16}(125\pi)^{1/2} x^2 \underbrace{\left(hc/2\pi G m_p^2\right)^{3/2}}_{N_0 = 2.2\cdot 10^{57}}$$

$$N_c = 0.7\left(\frac{x}{0.5}\right)^2 N_0 \Rightarrow N_c m_p = 1.4 M_\odot$$

numerically exact result called "Chandrasekhar limiting mass" as the largest mass a white dwarf can have and still cool off to a stable cold state with finite radius and density

Neutron Starts Stabilized by Fermi Pressure of Neutrons

$e^- + p \rightarrow n + \nu$ E(R) could be lowered still further by changing x through inverse beta decay requiring electrons with large kinetic energy

$$m_p \left(\frac{8\pi}{3x}\right)\left(\frac{m_e c}{h}\right)^3 = \frac{0.97}{x} \times 10^6 \text{ g/cm}^3 \ll 10^{11} \text{ g/cm}^3 < \rho < m_p \left(\frac{8\pi}{3(1-x)}\right)\left(\frac{m_n c}{h}\right)^3 = \frac{6}{1-x} \times 10^{15} \text{ g/cm}^3$$

density of star at which Fermi momentum of electrons equals $m_e c$

neutrons remain non-relativistic below this density

neutrons become more abundant than protons at these densities

$$E(R,x) = E_{\text{neutron}} + E_{\text{electron}} + E_{\text{gravitation}}$$

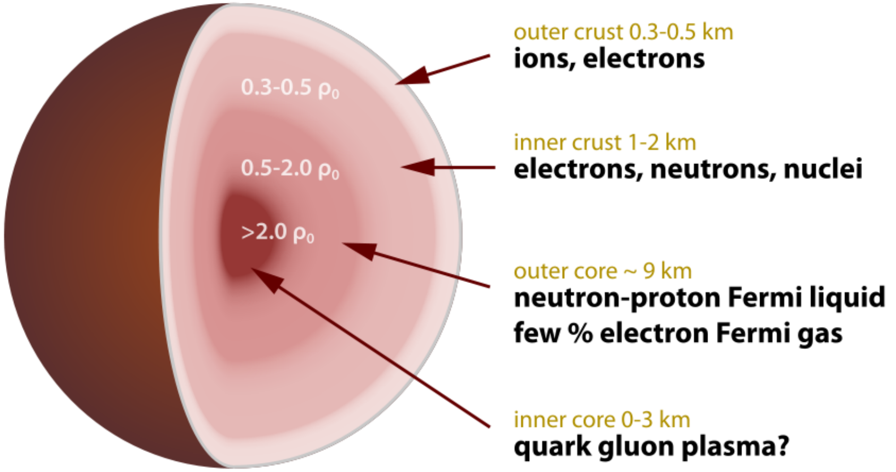
neglects energy of protons (since they are small fraction of nucleons in neutron star) and nuclear forces

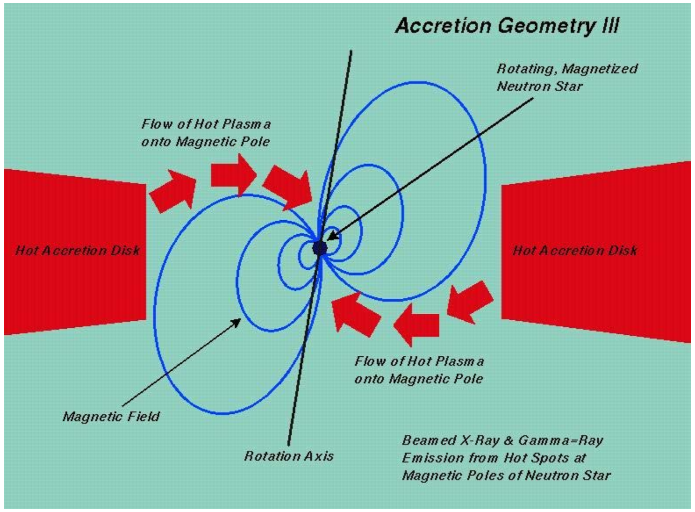
$$= \frac{3}{5}\frac{(1-x)Nh^2}{R^2 8m_p}\left(\frac{9}{4\pi^2}(1-x)N\right)^{2/3} + \frac{3}{8}\frac{xNhc}{R}\left(\frac{9}{4\pi^2}xN\right)^{1/3} - \frac{3GN^2 m_p^2}{5R}$$

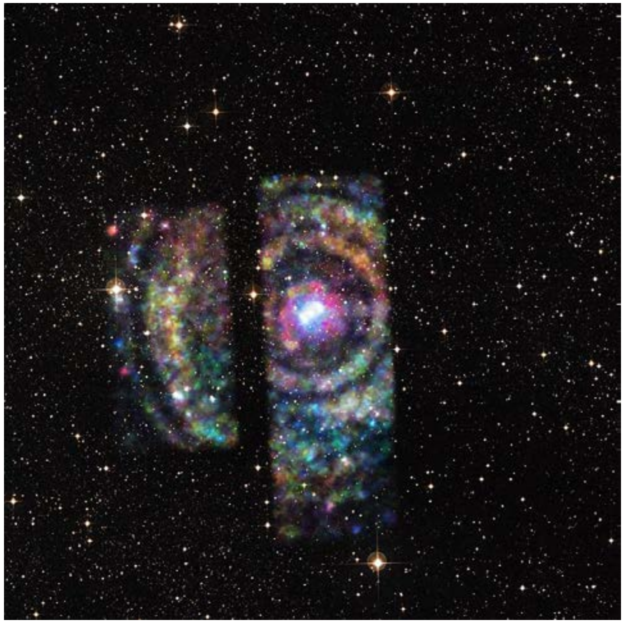
$$\frac{\partial}{\partial R}E(R,x) = 0; \frac{\partial}{\partial x}E(R,x) = 0 \Rightarrow \begin{cases} x = 0 \Rightarrow R = \dfrac{h^2}{4m_p}\left(\dfrac{9}{4\pi^2}N\right)^{2/3}\left(GNm_p^2\right)^{-1} \\ M = M_\odot; R = 12.6 \text{ km}; \rho = 2.4\times 10^{14} \text{ g/cm}^3 \Rightarrow x = 0.5\% \end{cases}$$

since it it composed 1-x=99.5% of neutrons, such star is like a giant atomic nucleus

Observing Pulsar-type of Neutron Stars Through Their Radiation







NASA Chandra satelite imaging of rings created by the X-rays from a Circinus-X1 double star system, containing neutron star orbiting around another massive star, reflecting off different dust clouds

Quark Deconfinement in High-Density Matter Neutron Star

$n \rightarrow 2d + u$ neutron density > 10 × 0.15 fm^{-3}

$\mu_n = 2\mu_d + \mu_u$ chemical equilibrium

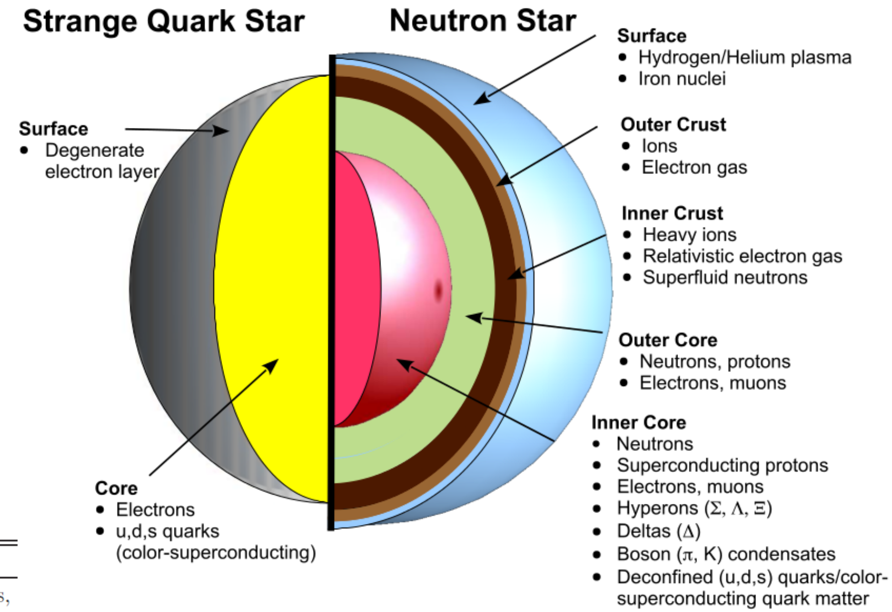
$\frac{2}{3}n_u - \frac{1}{3}n_d = 0 \Rightarrow n_u = \frac{1}{2}n_d$ charge conserved

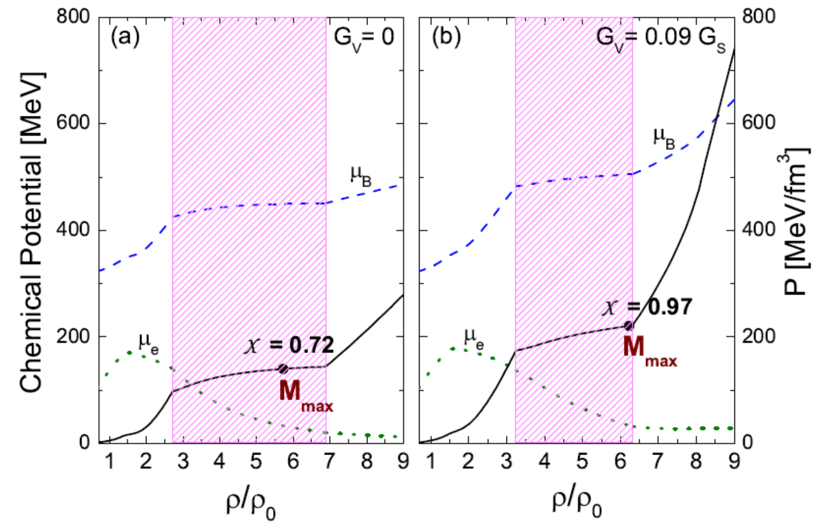
$P_n = P_d + P_u$ pressure equilibrium

At the density and pressure at which a neutron–quark phase transition is expected to occur, the system of neutrons is not ideal degenerate Fermi gas

Table I: Theoretical properties of strange quark stars and neutron stars compared.

Strange Quark Stars	Neutron Stars
Made entirely of deconfined up, down, strange quarks, and electrons	Nucleons, hyperons, boson condensates, deconfined quarks, electrons, and muons
Absent	Superfluid neutrons
Absent	Superconducting protons
Color superconducting quarks	Color superconducting quarks
Energy per baryon $\lesssim$ 930 MeV	Energy per baryon > 930 MeV
Self-bound ($M \propto R^3$)	Bound by gravity
Maximum mass $\sim 2\,M_\odot$	Same
No minimum mass if bare	Minimum mass $\sim 0.1\,M_\odot$
Radii $R \lesssim 10 - 12$ km	Radii $R \gtrsim 10 - 12$ km
Baryon numbers $B \lesssim 10^{57}$	Baryon numbers $10^{56} \lesssim B \lesssim 10^{57}$
Electric surface fields $\sim 10^{18}$ to $\sim 10^{19}$ V/cm	Absent
Can either be bare or enveloped in thin nuclear crusts (masses $\lesssim 10^{-5}\,M_\odot$)	Always have nuclear crusts
Maximum density of crust set by neutron drip, i.e., strange stars posses only outer crusts	Does not apply, i.e., neutron stars posses inner and outer crusts
Form two-parameter stellar sequences	Form one-parameter stellar sequences





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