

# Spin Torque in Spin Valve Nanopillars, Magnetic Tunnel Junctions and Spin-Orbit-Coupled Heterostructures

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<http://wiki.physics.udel.edu/phys824>



# References

Journal of Magnetism and Magnetic Materials 320 (2008) 1190–1216

## Current Perspectives Spin transfer torques

D.C. Ralph<sup>a,\*</sup>, M.D. Stiles<sup>b</sup>

### Abstract

This tutorial article introduces the physics of spin transfer torques in magnetic devices. We provide an elementary discussion of the mechanism of spin transfer torque, and review the theoretical and experimental progress in this field. **Our intention is to be accessible to beginning graduate students.** This is the introductory paper for a cluster of “Current Perspectives” articles on spin transfer torques published in volume **320** of the *Journal of Magnetism and Magnetic Materials*. This article is meant to set the stage for the others which follow it in this cluster; they focus in more depth on particularly interesting aspects of spin-torque physics and highlight unanswered questions that might be productive topics for future research.

REVIEW ARTICLES | INSIGHT

PUBLISHED ONLINE: 23 APRIL 2012 | DOI: 10.1038/NMAT3311

nature  
materials

### Current-induced torques in magnetic materials

Arne Brataas<sup>1</sup>, Andrew D. Kent<sup>2</sup> and Hideo Ohno<sup>3,4</sup>

nature  
materials

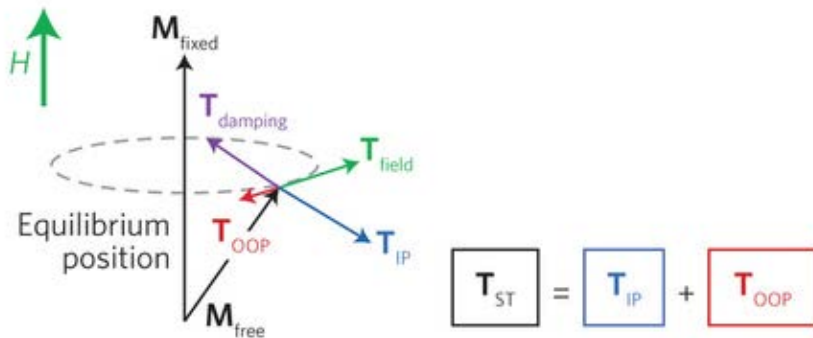
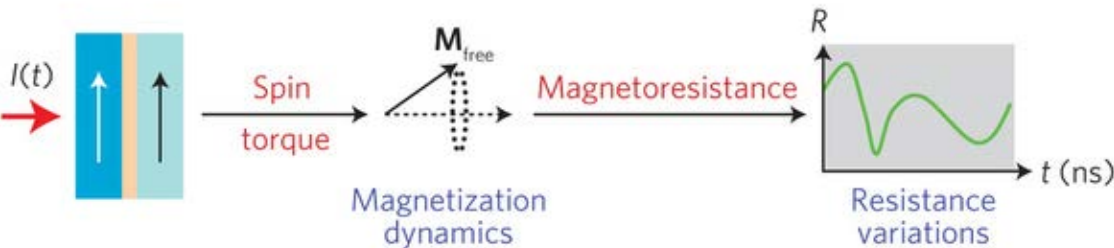
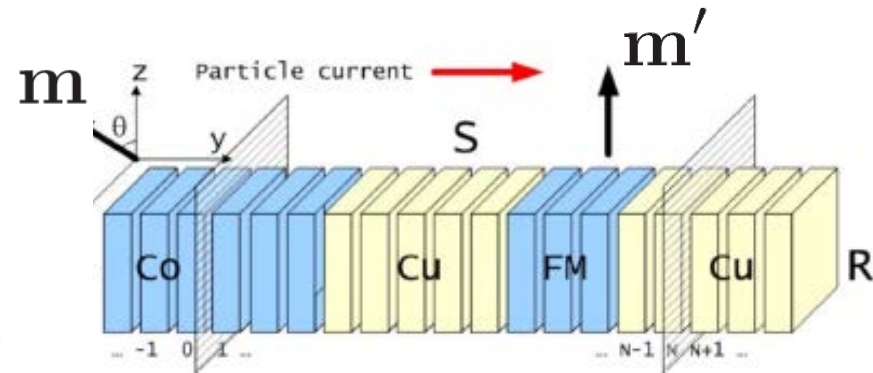
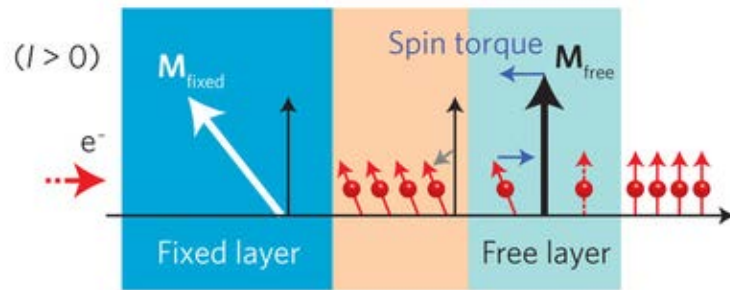
PROGRESS ARTICLE

PUBLISHED ONLINE: 17 DECEMBER 2013 | DOI: 10.1038/NMAT3823

### Spin-torque building blocks

N. Locatelli, V. Cros and J. Grollier\*

# Spin-Transfer Torque in Pictures and Basic Terminology



$$\mathbf{T} = \mathbf{T}_{\parallel} + \mathbf{T}_{\perp}$$

$$\mathbf{T}_{\parallel} = \tau_{\parallel} \mathbf{m} \times (\mathbf{m} \times \mathbf{m}')$$

$$\mathbf{T}_{\perp} = \tau_{\perp} \mathbf{m} \times \mathbf{m}'$$

## Terminology:

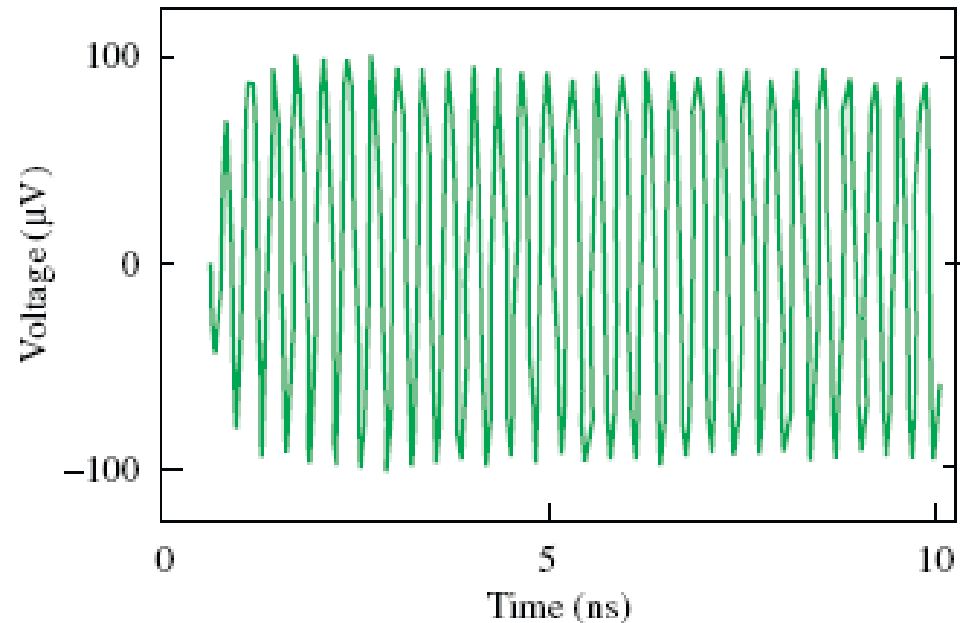
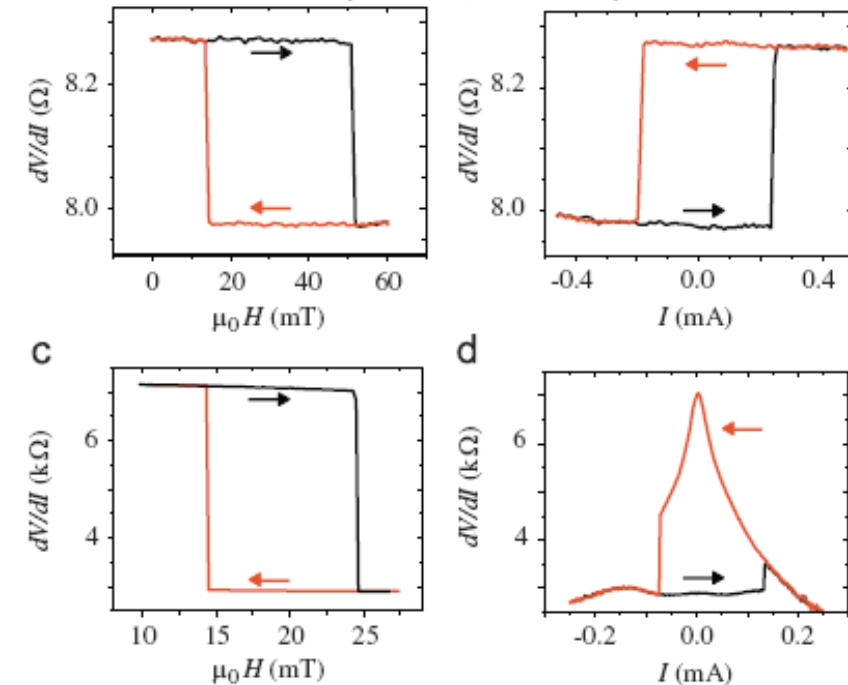
- In-plane or (anti) damping torque
- Parallel or field-like torque

# What Are Experimental Manifestations of STT?

## Magnetization Switching:

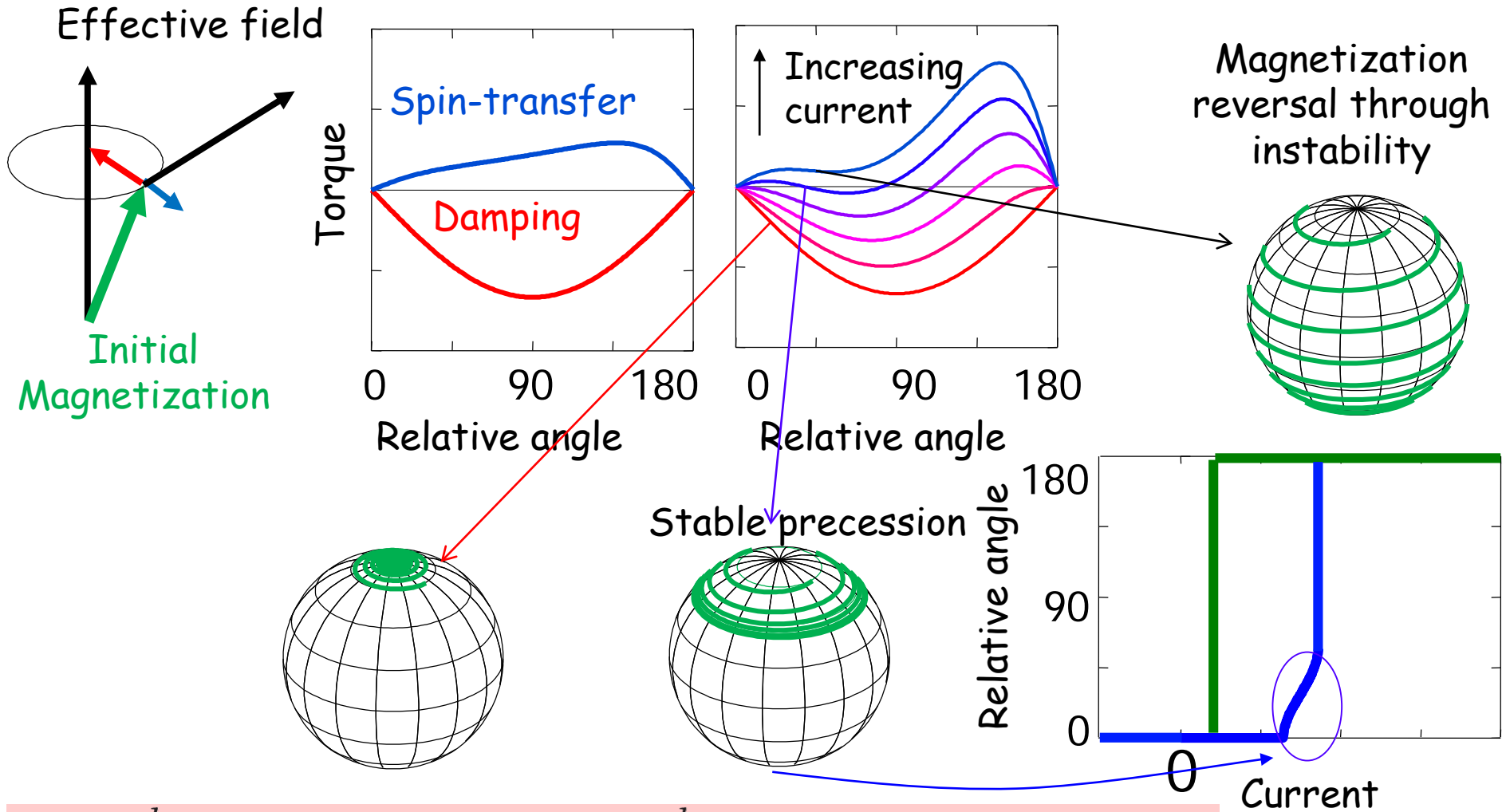
## Magnetization Precession:

20 nm Ni<sub>81</sub>Fe<sub>19</sub> / 12 nm Cu / 4.5 nm Ni<sub>81</sub>Fe<sub>19</sub>.



junction nanopillar sample consisting of the layers 15 nm PtMn / 2.5 nm Co<sub>70</sub>Fe<sub>30</sub> / 0.85 nm Ru / 3 nm Co<sub>60</sub>Fe<sub>20</sub>B<sub>20</sub> / 1.25 nm MgO / 2.5 nm Co<sub>60</sub>Fe<sub>20</sub>B<sub>20</sub>, as the 2.5-nm Co<sub>60</sub>Fe<sub>20</sub>B<sub>20</sub> free layer is re-

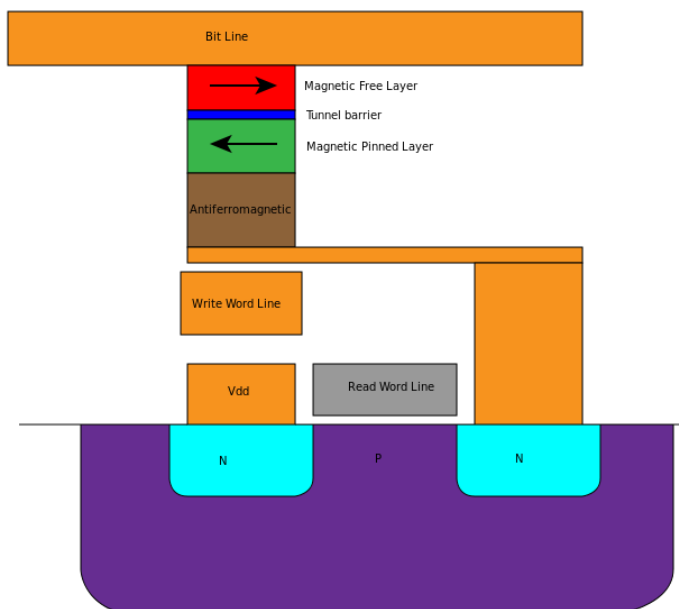
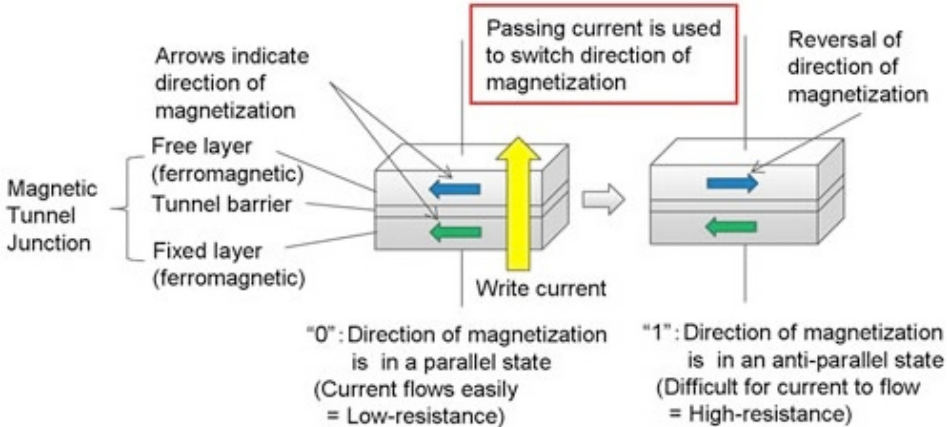
# Fast Quantum Electrons Interact with Slow Classical Magnetization Governed by LLG Equation



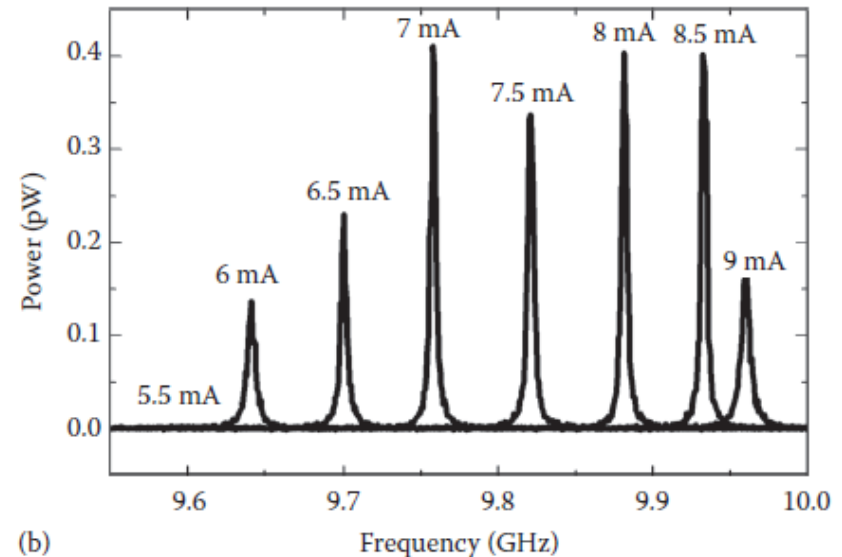
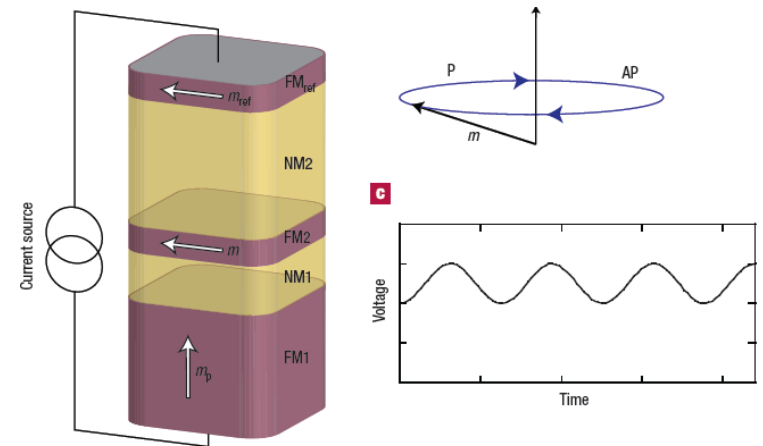
$$\text{LLG} : \frac{d\mathbf{m}}{dt} = -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt} + a_J \mathbf{m} \times (\mathbf{m} \times \mathbf{m}_{\text{polarizer}})$$

# Principal Applications of STT: STT-MRAM and STT-Nano-oscillators

## Non-volatile Memory:



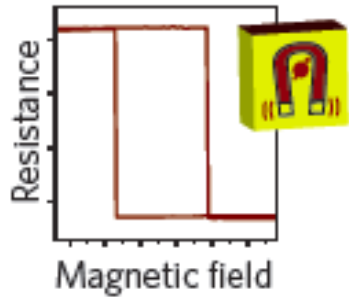
## Microwave Nano-Oscillators:



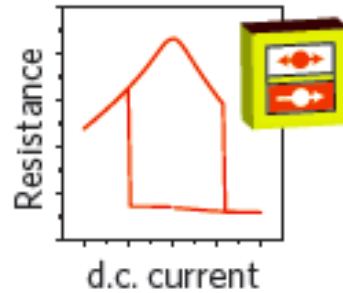
Nature Mater. 6, 447 (2007)

# Other Anticipated Technologies Based on STT

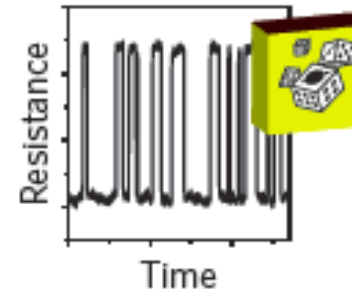
Detector (GMR, TMR)



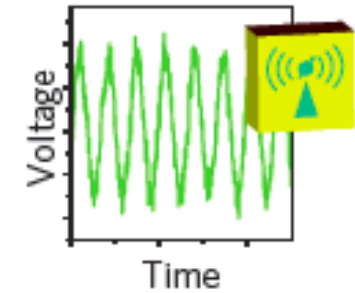
Binary memory



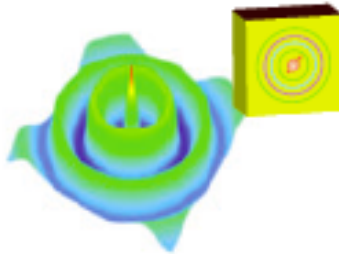
Stochastic device



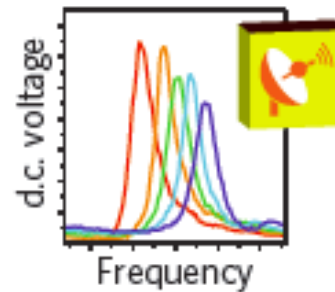
Microwave oscillator



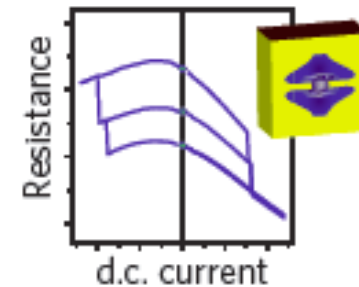
Spin-wave emitter



Microwave detector



Memristor



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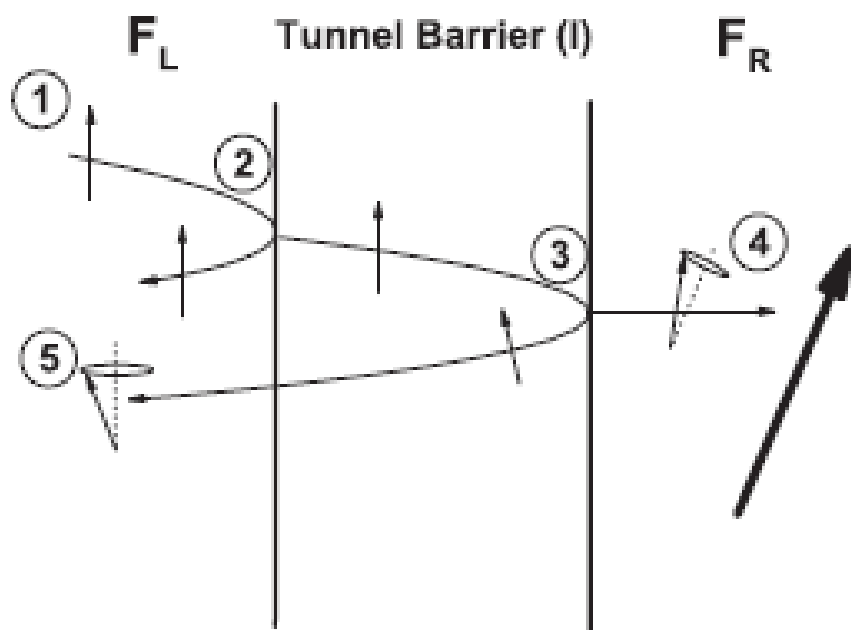
PUBLISHED ONLINE: 17 DECEMBER 2013 | DOI: 10.1038/NMAT3823

Spin-torque building blocks

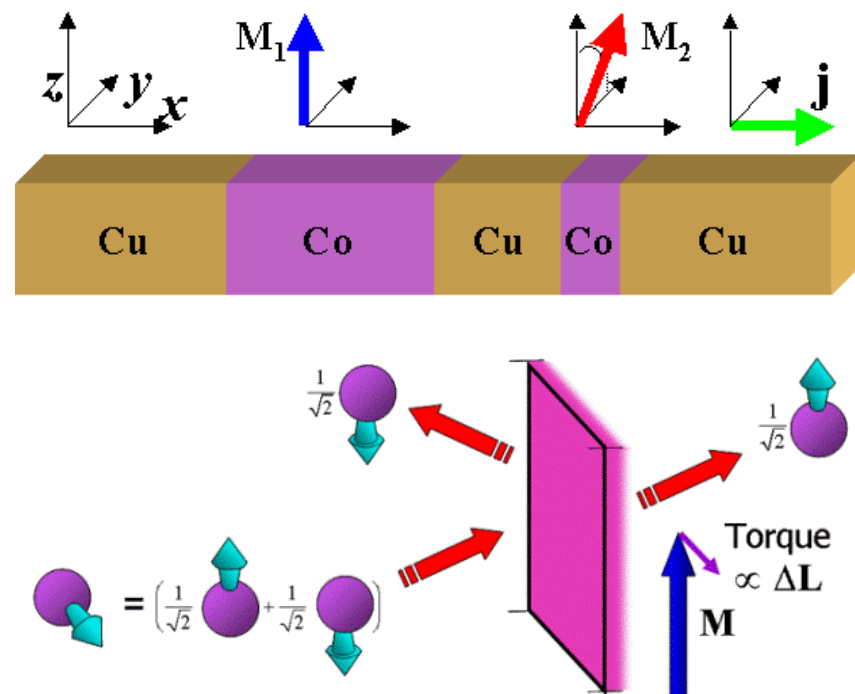
N. Locatelli, V. Cros and J. Grollier\*

# Physical Explanation of the Origin of STT

## Semiclassical:



## Fully Quantum:





# Undergrad Quantum-Mechanical Theory of STT: One-Dimensional Toy Model #1

$$\psi = \frac{e^{ikx}}{\sqrt{\Omega}} (a |\uparrow\rangle + b |\downarrow\rangle)$$

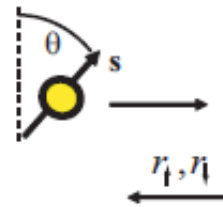
$$\mathbf{Q} = \frac{\hbar^2}{2m} \text{Im}(\psi^* \boldsymbol{\sigma} \otimes \nabla \psi)$$

$$Q_{xx} = \frac{\hbar^2 k}{2m\Omega} 2\text{Re}(ab^*)$$

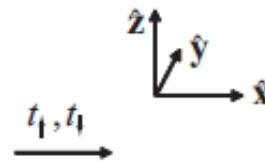
$$Q_{xy} = \frac{\hbar^2 k}{2m\Omega} 2\text{Im}(ab^*)$$

$$Q_{xz} = \frac{\hbar^2 k}{2m\Omega} (|a|^2 - |b|^2)$$

Toy model #1



M



$$\begin{aligned} N_{\text{st}} &= - \int_{\text{pillbox surfaces}} d^2 R \hat{n} \cdot \mathbf{Q} \\ &= - \int_{\text{pillbox volume}} d^3 r \nabla \cdot \mathbf{Q}, \end{aligned}$$

$$\begin{aligned} N_{\text{st}} &= A \hat{x} \cdot (\mathbf{Q}_{\text{in}} + \mathbf{Q}_{\text{refl}} - \mathbf{Q}_{\text{trans}}) \\ &= \frac{A \hbar^2 k}{\Omega 2m} \sin(\theta) \left[ 1 - \text{Re}(t_{\uparrow} t_{\downarrow}^* + r_{\uparrow} r_{\downarrow}^*) \right] \hat{x} \\ &\quad - \frac{A \hbar^2 k}{\Omega 2m} \sin(\theta) \text{Im}(t_{\uparrow} t_{\downarrow}^* + r_{\uparrow} r_{\downarrow}^*) \hat{y}. \end{aligned}$$

$$\mathbf{Q}_{\text{in}} = \frac{\hbar^2 k}{2m\Omega} \left[ \sin(\theta) \hat{x} + \cos(\theta) \hat{z} \right]$$

$$\begin{aligned} \mathbf{Q}_{\text{trans}} &= \frac{\hbar^2 k}{2m\Omega} \sin(\theta) \text{Re}(t_{\uparrow} t_{\downarrow}^*) \hat{x} \\ &\quad + \frac{\hbar^2 k}{2m\Omega} \sin(\theta) \text{Im}(t_{\uparrow} t_{\downarrow}^*) \hat{y} \\ &\quad + \frac{\hbar^2 k}{2m\Omega} \left[ |t_{\uparrow}|^2 \cos^2(\theta/2) - |t_{\downarrow}|^2 \sin^2(\theta/2) \right] \hat{z} \end{aligned}$$

$$\mathbf{Q}_{\text{refl}} = -\frac{\hbar^2 k}{2m\Omega} \sin(\theta) \text{Re}(r_{\uparrow} r_{\downarrow}^*) \hat{x}$$

$$-\frac{\hbar^2 k}{2m\Omega} \sin(\theta) \text{Im}(r_{\uparrow} r_{\downarrow}^*) \hat{y}$$

$$-\frac{\hbar^2 k}{2m\Omega} \left[ |r_{\uparrow}|^2 \cos^2(\theta/2) - |r_{\downarrow}|^2 \sin^2(\theta/2) \right] \hat{z}$$

$$\psi_{\text{in}} = \frac{e^{ikx}}{\sqrt{\Omega}} \left( \cos(\theta/2) |\uparrow\rangle + \sin(\theta/2) |\downarrow\rangle \right)$$

$$\psi_{\text{trans}} = \frac{e^{ikx}}{\sqrt{\Omega}} \left( t_{\uparrow} \cos(\theta/2) |\uparrow\rangle + t_{\downarrow} \sin(\theta/2) |\downarrow\rangle \right)$$

$$\psi_{\text{refl}} = \frac{e^{-ikx}}{\sqrt{\Omega}} \left( r_{\uparrow} \cos(\theta/2) |\uparrow\rangle + r_{\downarrow} \sin(\theta/2) |\downarrow\rangle \right)$$

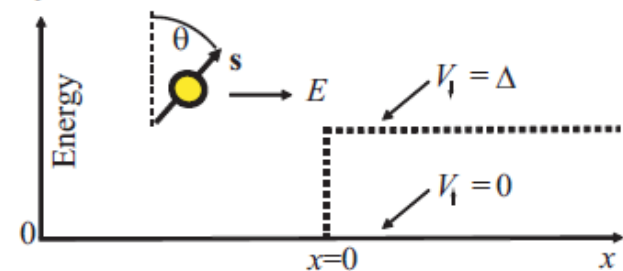
# Undergrad Quantum-Mechanical Theory of STT: One-Dimensional Toy Model #2

$$\psi_{\text{trans}} = \frac{e^{ik_{\uparrow}x}}{\sqrt{\Omega}} \cos(\theta/2) |\uparrow\rangle + \frac{e^{ik_{\downarrow}x}}{\sqrt{\Omega}} \frac{2k}{k+k_{\downarrow}} \sin(\theta/2) |\downarrow\rangle$$

$$\psi_{\text{refl}} = \frac{e^{-ikx}}{\sqrt{\Omega}} \frac{k-k_{\downarrow}}{k+k_{\downarrow}} \sin(\theta/2) |\downarrow\rangle,$$

$$k_{\uparrow} = k \text{ and } k_{\downarrow} = [2m(E - \Delta)]^{1/2}/\hbar < k$$

Toy model #2



$$\mathbf{Q} = \frac{\hbar^2}{2m} \text{Im}(\psi^* \boldsymbol{\sigma} \otimes \nabla \psi)$$

$$\mathbf{Q}_{\text{in}} = \frac{\hbar^2}{2m\Omega} (k \sin(\theta) \hat{x} + k \cos(\theta) \hat{z})$$

$$\mathbf{Q}_{\text{trans}} = \frac{\hbar^2}{2m\Omega} \sin(\theta) k \cos[(k_{\uparrow} - k_{\downarrow})x] \hat{x}$$

$$- \frac{\hbar^2}{2m\Omega} \sin(\theta) k \sin[(k_{\uparrow} - k_{\downarrow})x] \hat{y}$$

$$+ \frac{\hbar^2}{2m\Omega} \left[ k \cos^2(\theta/2) - k_{\downarrow} \left( \frac{2k}{k+k_{\downarrow}} \right)^2 \sin^2(\theta/2) \right] \hat{z}$$

$$\mathbf{Q}_{\text{refl}} = \frac{\hbar^2}{2m\Omega} k \left( \frac{k-k_{\downarrow}}{k+k_{\downarrow}} \right)^2 \sin^2(\theta/2) \hat{z}.$$

$$\mathbf{N}_{\text{st}} = A \hat{x} \cdot (\mathbf{Q}_{\text{in}} + \mathbf{Q}_{\text{refl}} - \mathbf{Q}_{\text{trans}}) \approx A \hat{x} \cdot \mathbf{Q}_{\text{in}}$$

$$\mathbf{N}_{\text{st}} \approx \frac{A \hbar^2 k}{\Omega 2m} \sin(\theta) \hat{x}$$

when summing or averaging over all contributions from around the Fermi surface, dephasing leads to  $Q_{\text{refl}} \approx 0$ ,  $Q_{\text{trans}} \approx 0$  (to a good approximation valid for typical metallic interfaces), so that STT acting on the magnet per unit area being equal to the full component of incident spin current that is transverse to magnetization of a ferromagnet

JMMM 320, 1190 (2008)

# Nonequilibrium Density Matrix for Steady-State Quantum Transport

□ Equilibrium density matrix is universal (fixed by Boltzmann and Gibbs):

$$\hat{\rho}_{\text{eq}} = \frac{e^{-\beta\hat{H}}}{\text{Tr} e^{-\beta\hat{H}}} \Rightarrow \langle \hat{A} \rangle = \text{Tr} [\hat{\rho}_{\text{eq}} \hat{A}]$$

□ Applied to non-interacting fermions in equilibrium:

$$\hat{\rho}_{\text{eq}} = \sum_{\alpha} f(E_{\alpha}) |E_{\alpha}\rangle \langle E_{\alpha}| \Leftrightarrow \hat{\rho}_{\text{eq}} = -\frac{1}{\pi} \int dE \text{Im}[\hat{G}_0^r(E)] f(E)$$
$$\hat{G}_0^r = [E - \hat{H} + i\eta]^{-1} \text{ or } \hat{G}_0^r = [E - \hat{H} - \hat{\Sigma}_L - \hat{\Sigma}_R]^{-1}$$

□ Equilibrium-like density matrix for steady-state transport of interacting fermions:

$$\hat{\rho}_{\text{neq}} \propto e^{-\beta(\hat{H} - \hat{Y})}$$

□ Nonequilibrium density matrix in terms of NEGFs:

$$\hat{\rho}_{\text{neq}} = \frac{1}{2\pi i} \int dE \hat{G}^<(E) - \frac{1}{\pi} \int dE \text{Im}[\hat{G}_0^r(E)] f(E)$$

# How to Remove Equilibrium Expectation Values in Gauge Invariant Fashion

□ Density matrix often split into "equilibrium" + "nonequilibrium" contributions for purely computational purposes:

$$\hat{\rho} = -\frac{1}{\pi} \int_{-\infty}^{+\infty} dE \operatorname{Im} \hat{G}^r(E) f(E - eV_R) + \frac{1}{2\pi} \int_{-\infty}^{+\infty} dE \hat{G}^r(E) \cdot \hat{\Gamma}_L(E - eV_L) \cdot \hat{G}^a(E) [f(E - eV_L) - f(E - eV_R)]$$

□ The proper gauge-invariant nonequilibrium density matrix is defined by:

$$\hat{\rho}_{\text{neq}} = \hat{\rho} - \hat{\rho}_{\text{eq}} = \hat{\rho} + \frac{1}{\pi} \int_{-\infty}^{+\infty} dE \operatorname{Im} \left[ \hat{G}_0^r(E) \right] f(E)$$

□ First two terms below remove any equilibrium contribution to physical quantity whose non-zero value is compatible with time-reversal invariance (zero T limit):

$$\begin{aligned} \hat{\rho}_{\text{neq}} = & -\frac{eV_R}{\pi} \operatorname{Im} [G_0^r(E_F)] - \frac{1}{\pi} \int_{-\infty}^{E_F} dE \operatorname{Im} \left[ \hat{G}_0^r \left( eU - eV_L \frac{\partial \hat{\Sigma}_L}{\partial E} - eV_R \frac{\partial \hat{\Sigma}_R}{\partial E} \right) \hat{G}_0^r \right] f(E) \\ & + \frac{eV_b}{2\pi} \hat{G}_0^r(E_F) \cdot \hat{\Gamma}_L(E_F) \cdot \hat{G}_0^a(E_F), \end{aligned}$$

SPIN 3, 1330002 (2013)

# Graduate Quantum-Mechanical Theory of STT using Torque Operator and NEGF Formulas

$$\hat{H} = \hat{H}_0 + \Delta(\mathbf{m} \cdot \boldsymbol{\sigma}) \Rightarrow \hat{\mathbf{T}} = \Delta(\mathbf{m} \times \boldsymbol{\sigma}) \Rightarrow \mathbf{T} = \text{Tr}[\hat{\rho}_{\text{neq}} \hat{\mathbf{T}}]$$

$$\hat{\rho}_{\text{neq}} = -\frac{eV_R}{\pi} \text{Im}[G_0^r(E_F)] - \frac{1}{\pi} \int_{-\infty}^{E_F} dE \text{Im} \left[ \hat{G}_0^r \left( eU - eV_L \frac{\partial \hat{\Sigma}_L}{\partial E} - eV_R \frac{\partial \hat{\Sigma}_R}{\partial E} \right) \hat{G}_0^r \right] f(E) + \frac{eV_b}{2\pi} \hat{G}_0^r(E_F) \cdot \hat{\Gamma}_L(E_F) \cdot \hat{G}_0^a(E_F)$$

$$\hat{\mathbf{T}} = \frac{d\hat{\mathbf{S}}}{dt} = -\frac{i}{\hbar} \left[ \frac{\hbar}{2} \boldsymbol{\sigma}, \hat{H} \right]$$

$$(\boldsymbol{\sigma} \cdot \mathbf{p})(\boldsymbol{\sigma} \cdot \mathbf{q}) = \mathbf{p} \cdot \mathbf{q} + i(\mathbf{p} \times \mathbf{q}) \cdot \boldsymbol{\sigma}$$

$$(\boldsymbol{\sigma} \cdot \mathbf{p})\boldsymbol{\sigma} = \mathbf{p} + i\boldsymbol{\sigma} \times \mathbf{p}$$

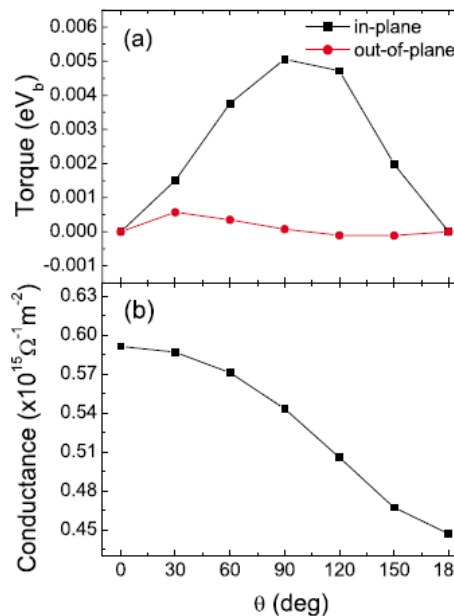
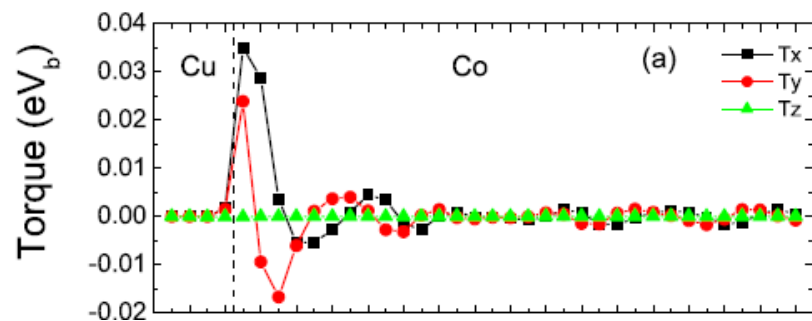
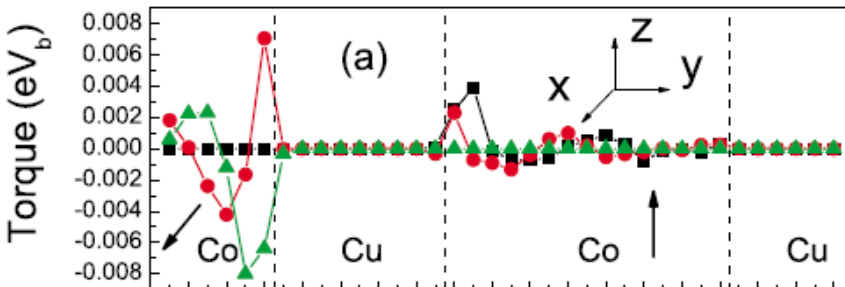
$$\boldsymbol{\sigma}(\boldsymbol{\sigma} \cdot \mathbf{q}) = \mathbf{q} + i\mathbf{q} \times \boldsymbol{\sigma}$$

$$\hat{\rho}_{\text{neq}} = \frac{1}{2\pi i} \int dE [\hat{G}^<(E, V_b) - \hat{G}^<(E, V_b = 0)]$$

$$\hat{G}^r = [E - \hat{H} - \hat{\Sigma}_L^r - \hat{\Sigma}_R^r]^{-1}$$

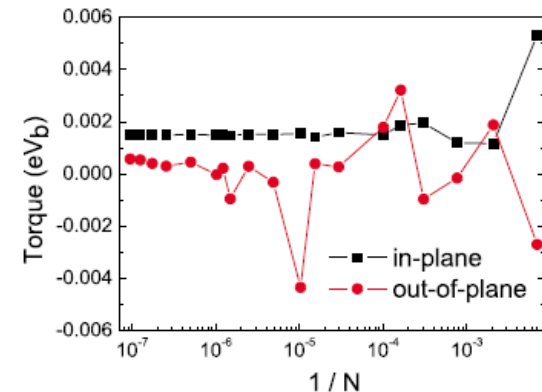
$$\hat{\Sigma}^< = if_L \hat{\Gamma}_L + if_R \hat{\Gamma}_R \text{ (for elastic transport)}$$

$$\hat{G}^< = \hat{G}^r \hat{\Sigma}^< \hat{G}^a$$

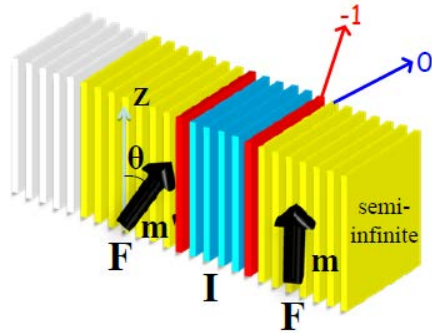


Co( $\theta$ )|Cu(9 ML)|Co(15 ML)|Cu

PRB 77, 184430 (2008)



# NEGF Formulas for STT in the Absence of Spin Flips Using Interfacial Spin Current

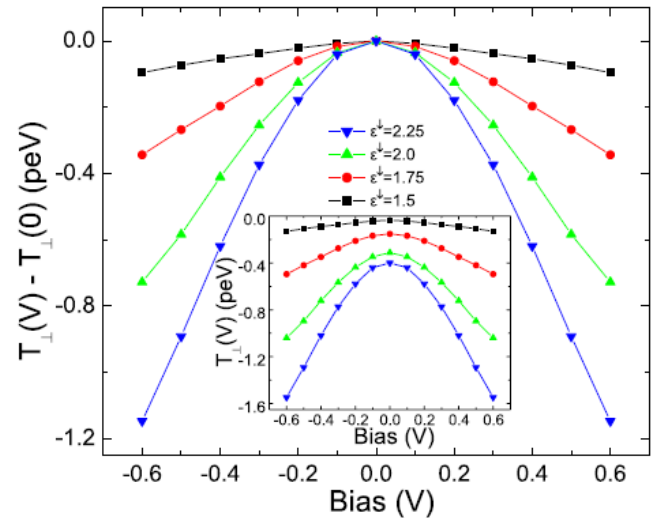
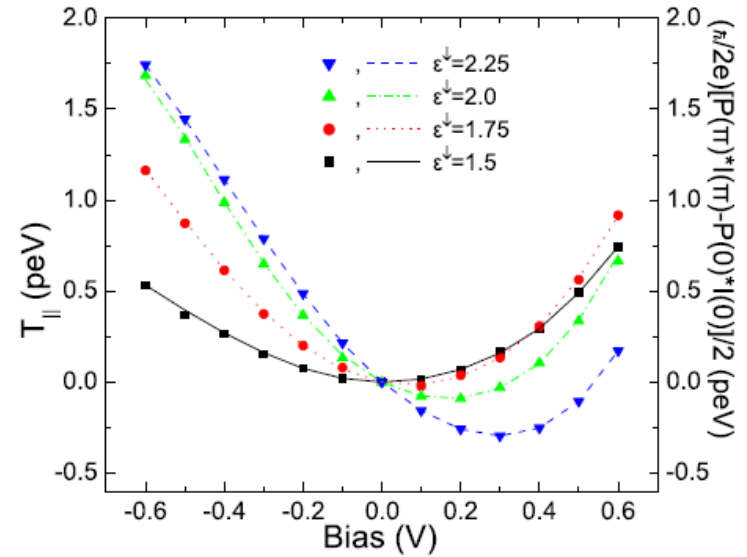
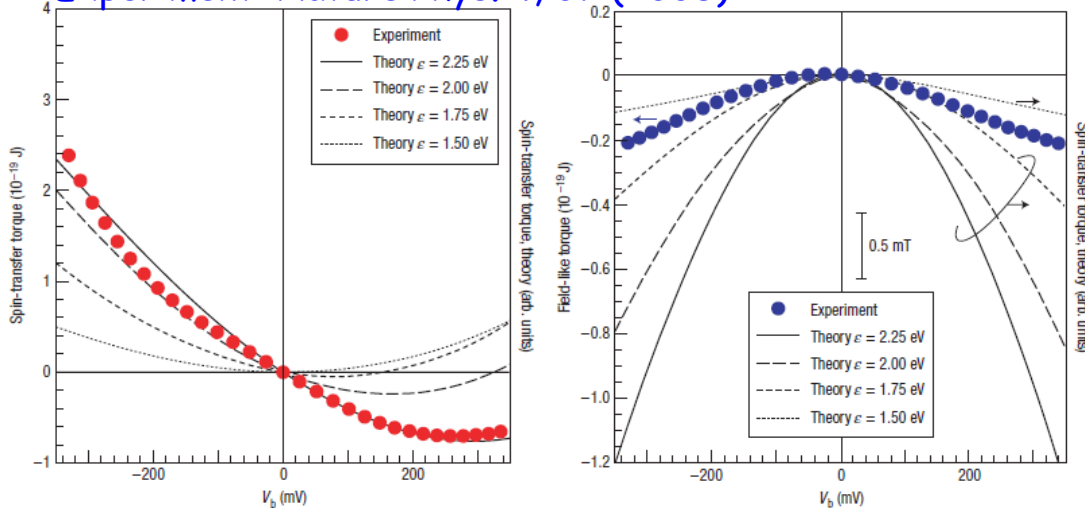


$$\mathbf{I}_{n,n+1}^S = \frac{\gamma}{4\pi} \int dE d\mathbf{k}_{\parallel} \text{Tr}_{\sigma} [\boldsymbol{\sigma} (G_{n+1,n}^{<\sigma,\sigma} - G_{n,n+1}^{<\sigma,\sigma'})]$$

$$\mathbf{T}_n = -\nabla \cdot \mathbf{I}^S = \mathbf{I}_{n-1,n}^S - \mathbf{I}_{n,n+1}^S$$

$$\mathbf{T} = \sum_{\lambda'=0}^{\infty} (\mathbf{I}_{n-1,n}^S - \mathbf{I}_{n,n+1}^S) = \mathbf{I}_{-1,0}^S - \mathbf{I}_{\infty,\infty}^S = \mathbf{I}_{-1,0}^S$$

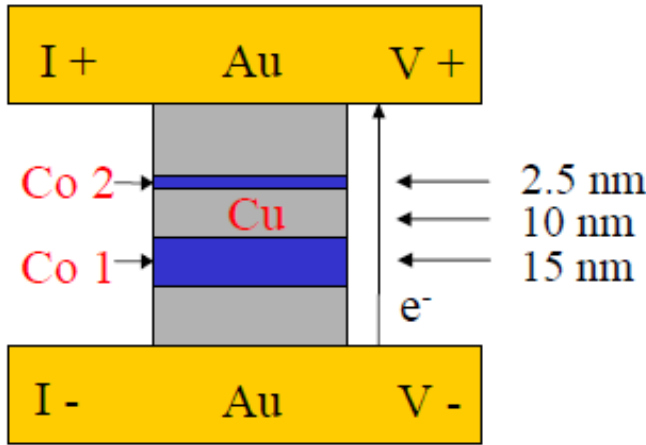
Experiment: Nature Phys. 4, 37 (2008)



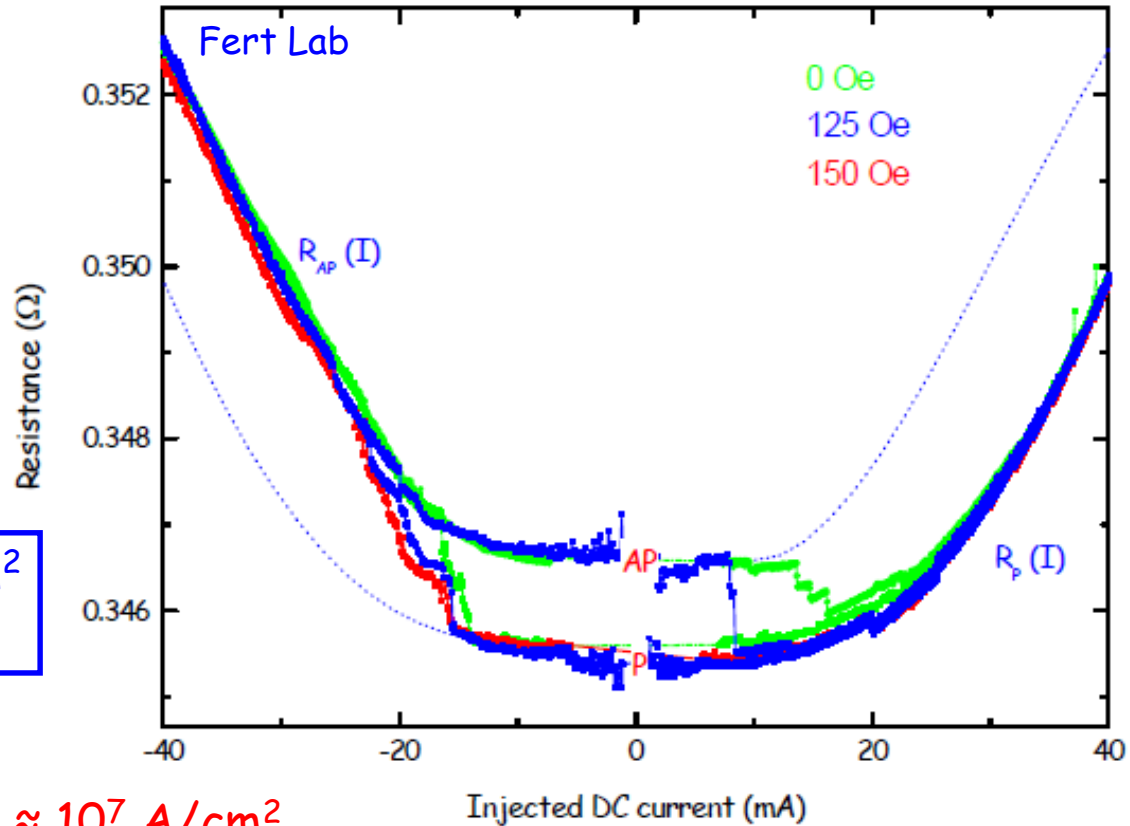
Theory: PRL 97, 237205 (2006)

# Experiments on STT in Spin Valves

nanopillar geometry

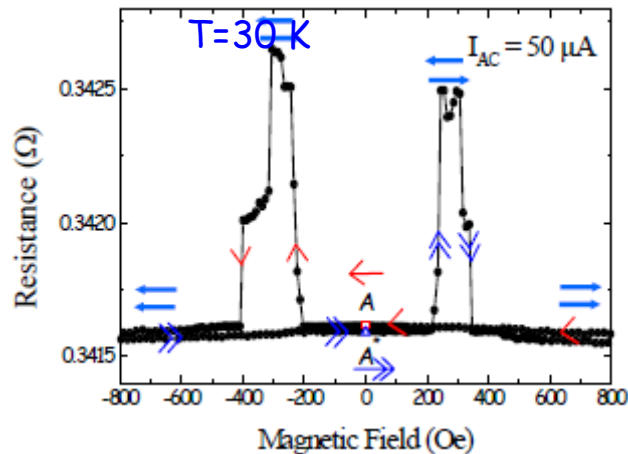


Low-impedance (RA)  $\sim 0.01 \Omega \mu\text{m}^2$   
GMR  $\sim 10\text{-}20\%$



$$|J_c| \approx 10^7 \text{ A/cm}^2$$

- Critical current densities quite similar in good spin valves and MTJs (high polarization of MTJs may give  $\sim 2x$  advantage)
- Conventional ferromagnet spin transfer devices require lateral dimensions  $\leq 250 \text{ nm}$  to avoid significant self-field effects from required current levels



# Experiments on STT in Magnetic Tunnel Junctions

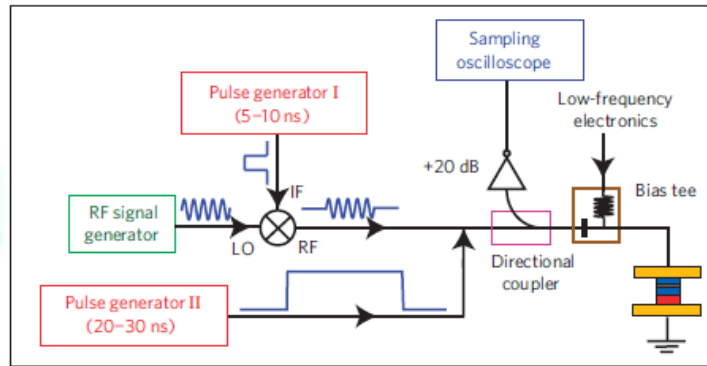
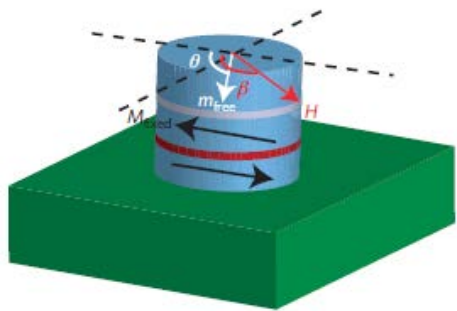
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PUBLISHED ONLINE: 27 FEBRUARY 2011 | DOI: 10.1038/NPHYS1928

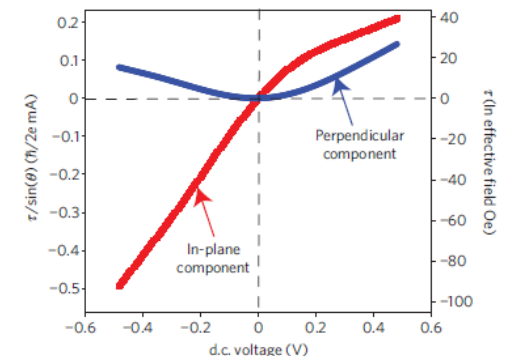
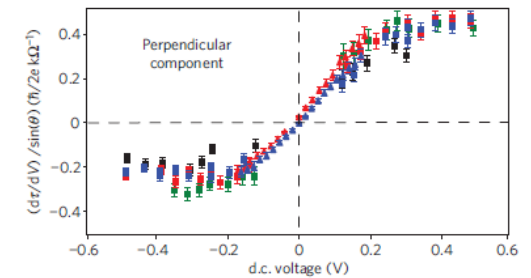
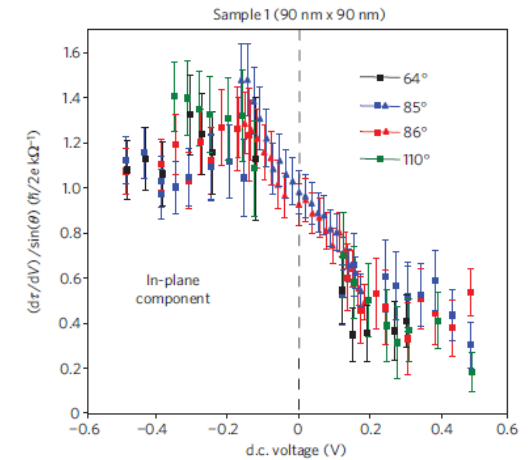
nature  
physics

## Time-resolved measurement of spin-transfer-driven ferromagnetic resonance and spin torque in magnetic tunnel junctions

Chen Wang<sup>1</sup>, Yong-Tao Cui<sup>1</sup>, Jordan A. Katine<sup>2</sup>, Robert A. Buhrman<sup>1</sup> and Daniel C. Ralph<sup>1,3\*</sup>



High-impedance (RA)  $\sim 1\text{-}100 \Omega\mu\text{m}^2$   
TMR  $\sim 100\%$





# New Frontier: Spin-Orbit Coupling-Driven STT on Single Ferromagnetic Layer

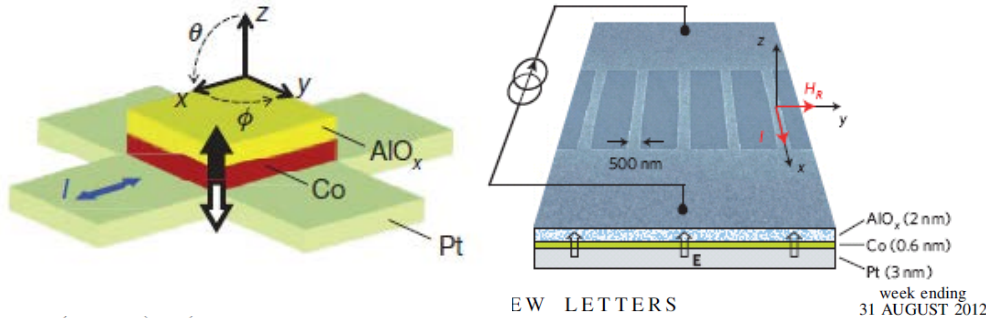
LETTERS

PUBLISHED ONLINE: 10 JANUARY 2010 | DOI: 10.1038/NMAT2613

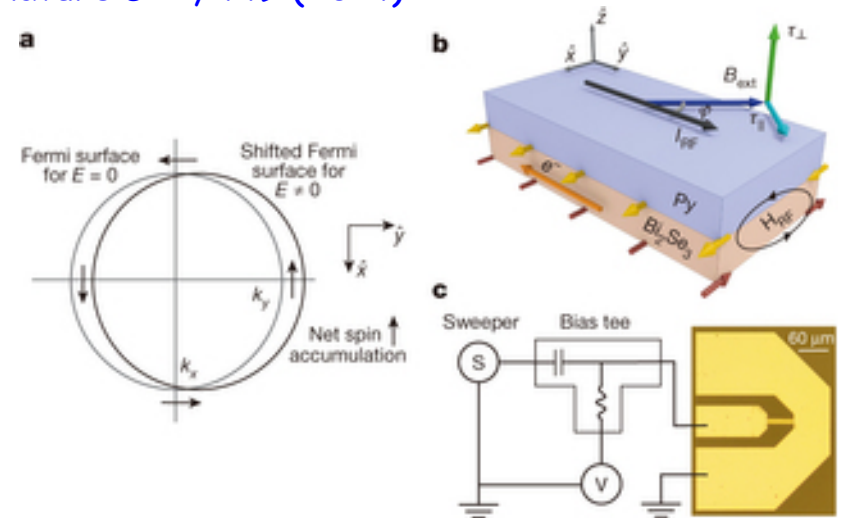
nature materials

## Current-driven spin torque induced by the Rashba effect in a ferromagnetic metal layer

Ioan Mihai Miron<sup>1\*</sup>, Gilles Gaudin<sup>2</sup>, Stéphane Auffret<sup>2</sup>, Bernard Rodmacq<sup>2</sup>, Alain Schuhl<sup>2</sup>, Stefania Pizzini<sup>3</sup>, Ian Mores<sup>3</sup> and Dietro Gambardella<sup>1,4</sup>



Nature 511, 449 (2014)

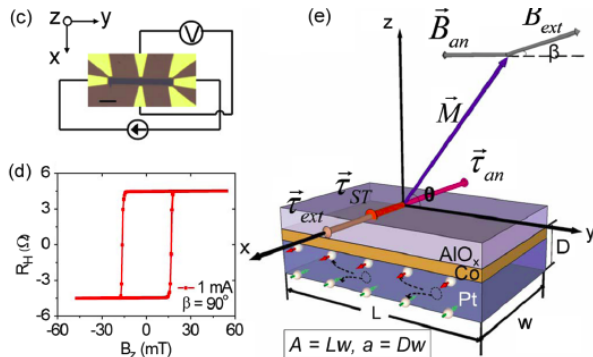


## Current-Induced Switching of Perpendicularly Magnetized Magnetic Layers Using Spin Torque from the Spin Hall Effect

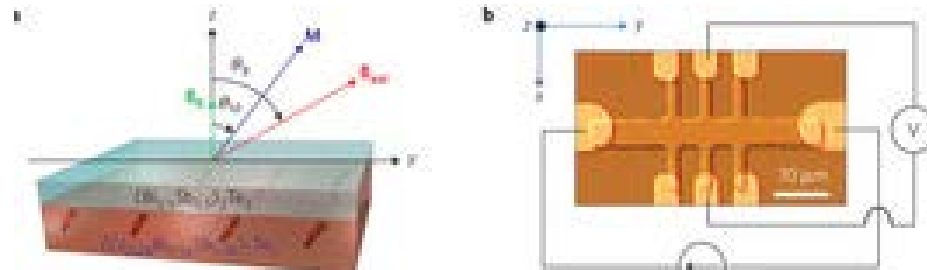
Luqiao Liu,<sup>1</sup> O. J. Lee,<sup>1</sup> T. J. Gudmundsen,<sup>1</sup> D. C. Ralph,<sup>1,2</sup> and R. A. Buhrman<sup>1</sup>

<sup>1</sup>Cornell University, Ithaca, New York 14853, USA

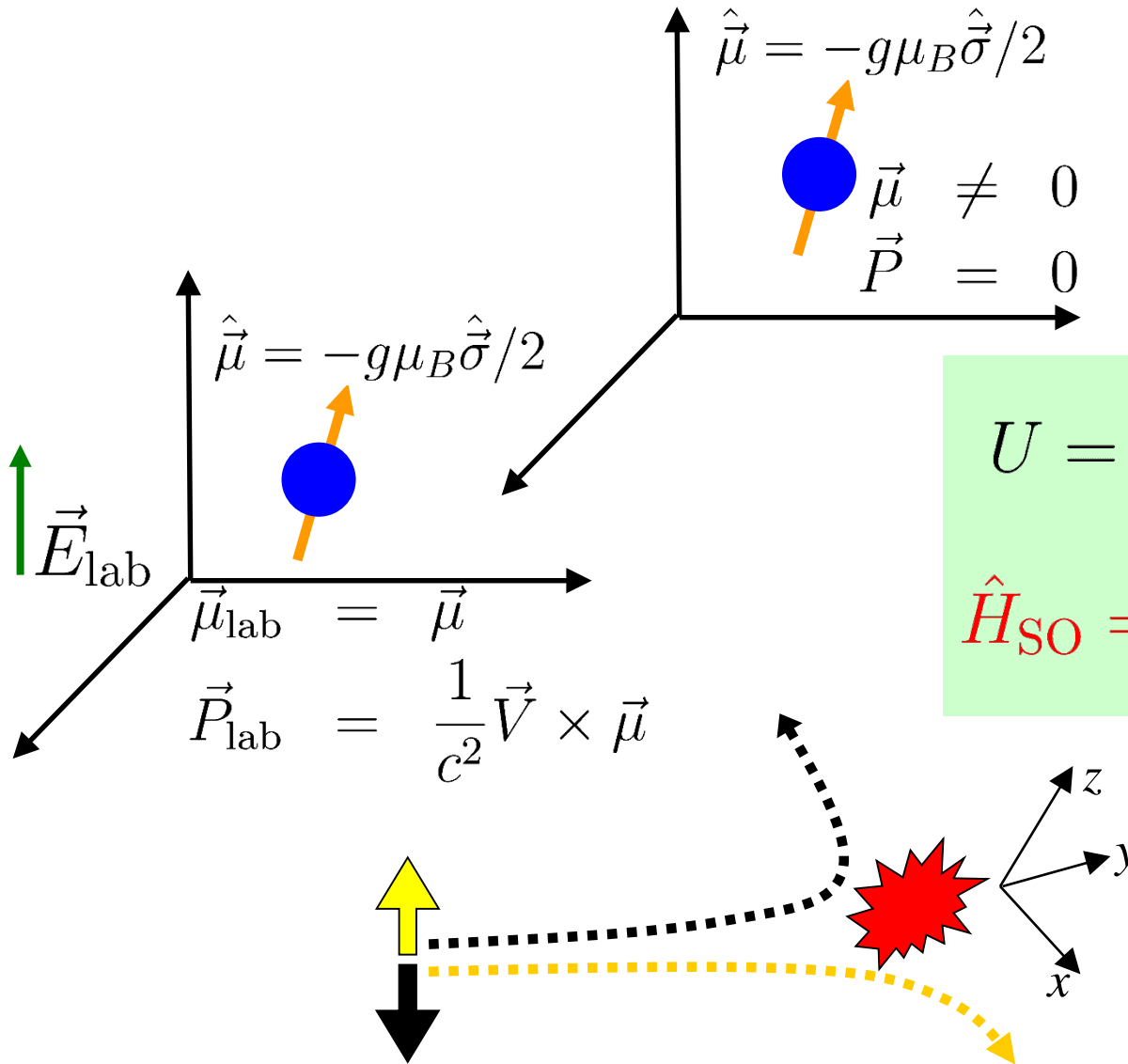
<sup>2</sup>Kavli Institute at Cornell, Ithaca, New York, 14853



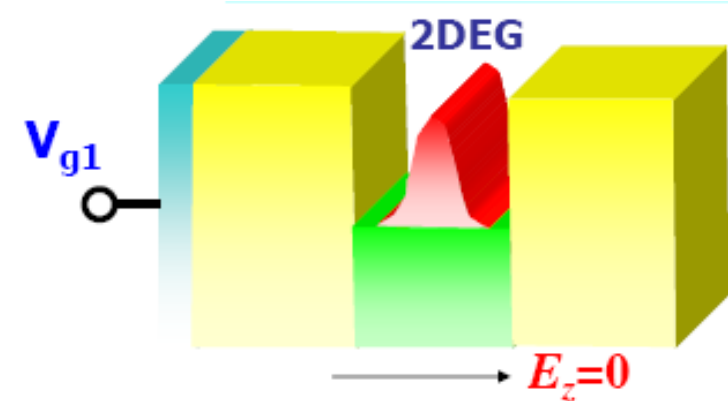
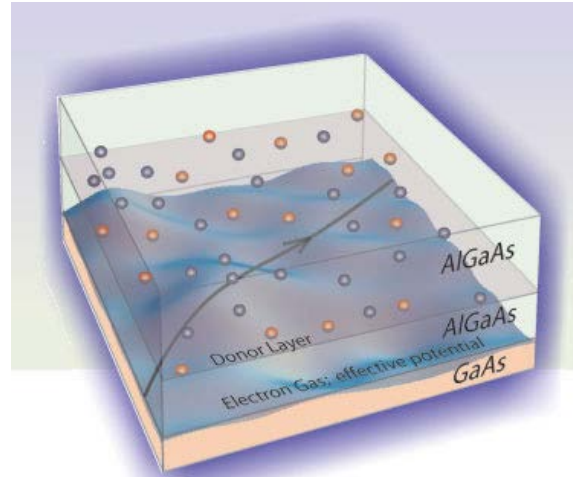
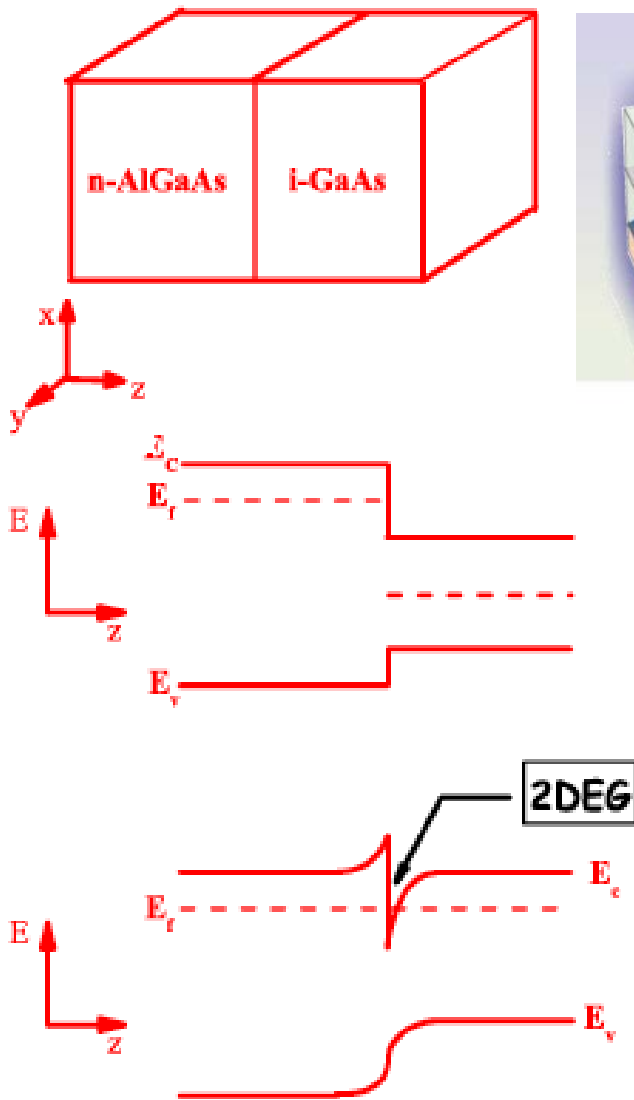
Nature 511, 449 (2014)



# What is Spin-Orbit Coupling?

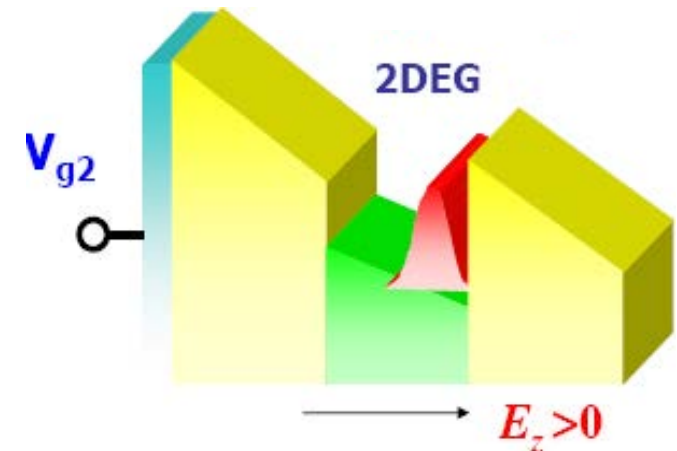


# Structural Inversion Asymmetry of 2DEGs in Semiconductor Heterostructures



**Inversion symmetry  $\Rightarrow$  no R-SO**

$$\begin{aligned} \varepsilon(\mathbf{k}, \uparrow) &= \varepsilon(-\mathbf{k}, \downarrow) \\ \varepsilon(\mathbf{k}, \uparrow) &\neq \varepsilon(-\mathbf{k}, \uparrow) \\ \varepsilon(\mathbf{k}, \uparrow) &\neq \varepsilon(\mathbf{k}, \downarrow) \end{aligned}$$



**Broken inversion symmetry  $\Rightarrow$  R-SO**

# Vacuum vs. Crystalline SO Coupling Strength

$$\hat{H}_{\text{Dirac}} = \beta m_0 c^2 + eV + c\boldsymbol{\alpha} \cdot (\hat{\mathbf{p}} - e\mathbf{A})$$

On the  $v^2/c^2$  expansion of the Dirac equation with external potentials

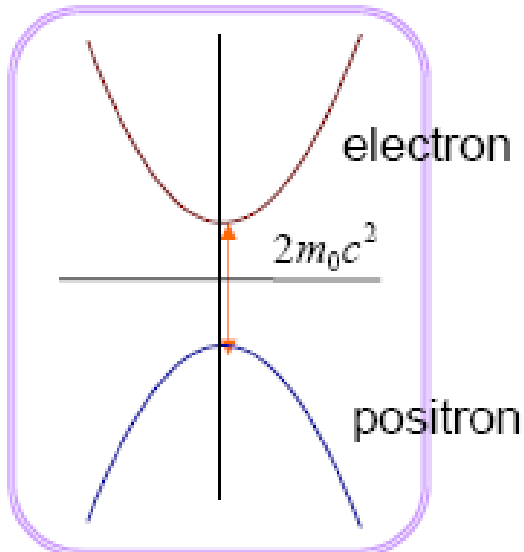
Wlodek Zawadzki<sup>\*)</sup>

Institute of Physics, Polish Academy of Sciences, Al. Lotnikow 32/46, 02-668 Warsaw, Poland

(Received 13 January 2005; accepted 8 April 2005)

The  $v^2/c^2$  expansion of the Dirac equation with external potentials is reexamined. A complete, gauge invariant form of the expansion to order  $(1/c)^2$  is established that contains two additional terms, in contrast to various published results. It is shown that the additional terms describe the relativistic decrease of the electron spin magnetic moment with increasing electron energy. © 2005 American Association of Physics Teachers.

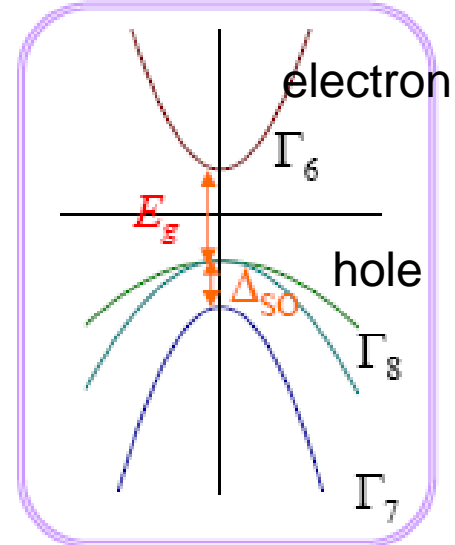
## VACUUM



Nonrelativistic expansion of the Dirac equation can be seen as a method of systematically including the effects of the negative energy solutions on the states of positive energy starting from their nonrelativistic limit

$$\hat{H}_{\text{SO}} = \frac{\hbar^2}{4m_0^2c^2} \hat{\boldsymbol{\sigma}} \cdot [\hat{\mathbf{k}} \times \nabla(V_{\text{nucleus}} + V_{\text{interface}})]$$

## SEMICONDUCTORS



$$\lambda_{\text{vac}} = \hbar^2 / 4m_0^2c^2$$

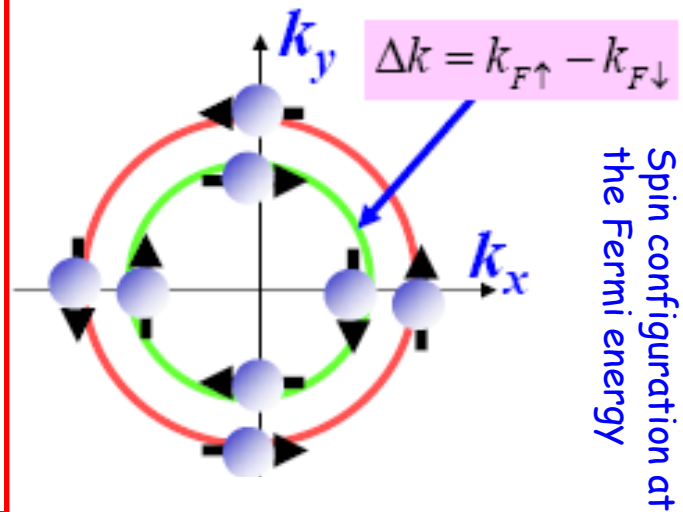
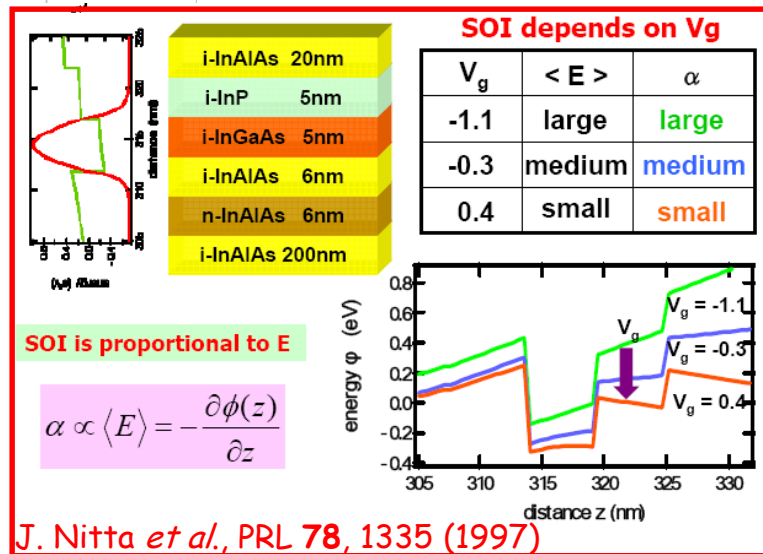
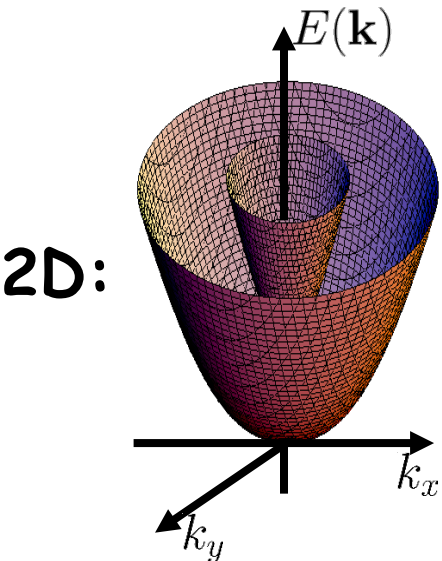
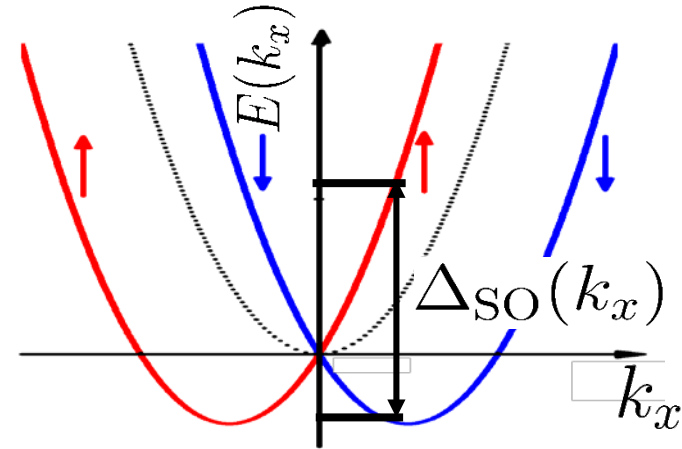
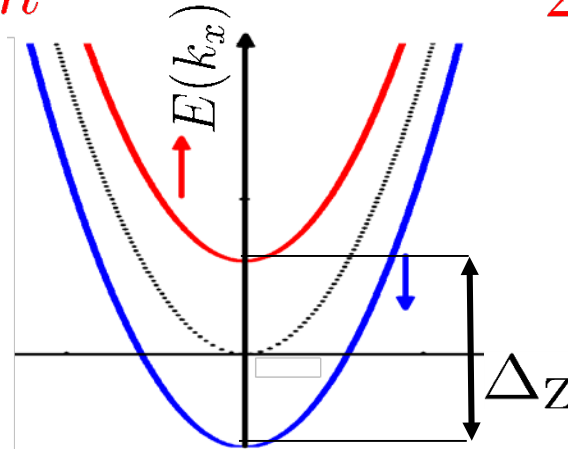
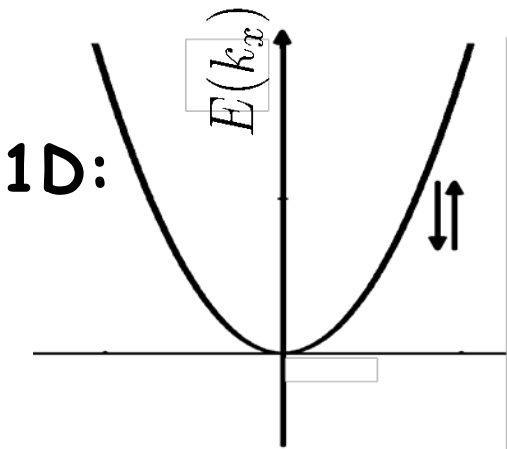
$$\lambda_{\text{vac}} = 3.7 \times 10^{-6} \text{Å}^2$$

$$\lambda_{\text{Sm}} = (P^2/3) [1/E_g^2 - 1/(E_g + \Delta_{\text{SO}})^2]$$

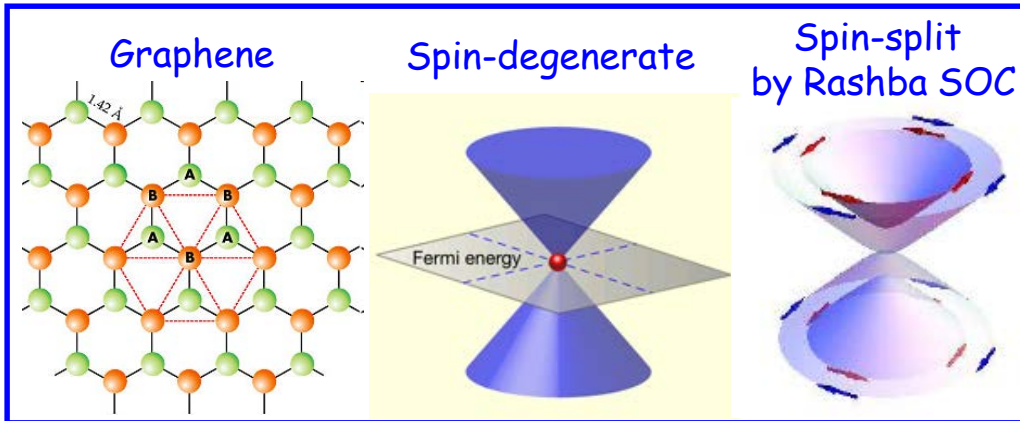
$$\lambda_{\text{GaAs}} = 5.3 \text{Å}^2$$

# Rashba SO Splitting of Energy Bands in 2DEGs

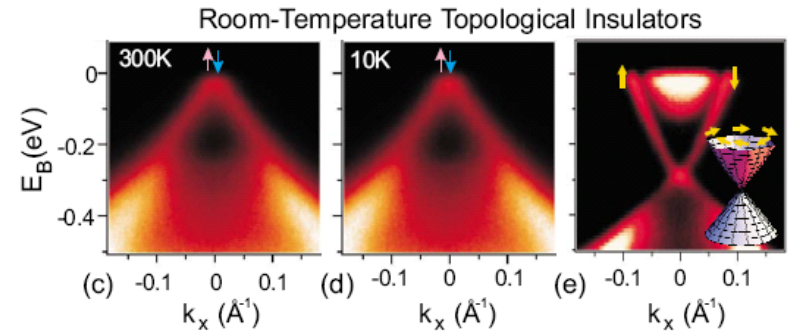
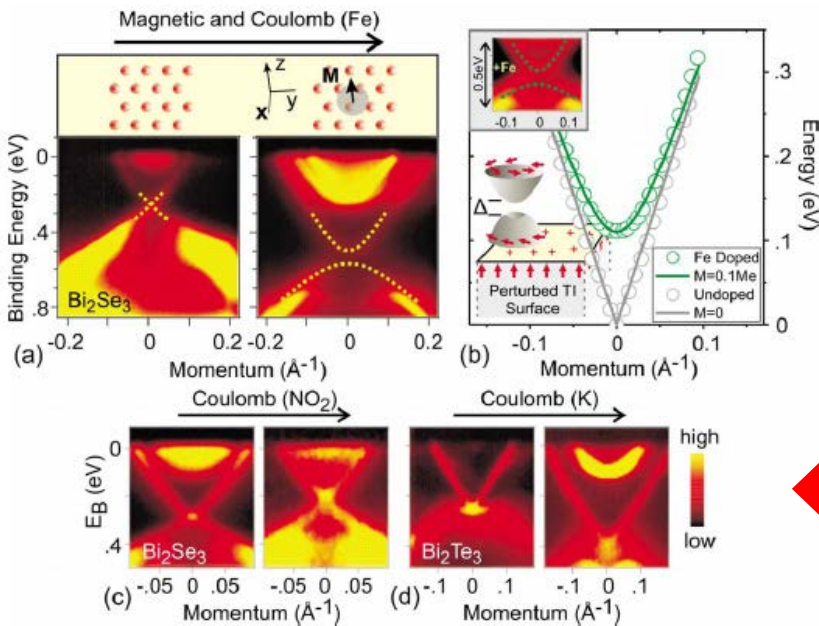
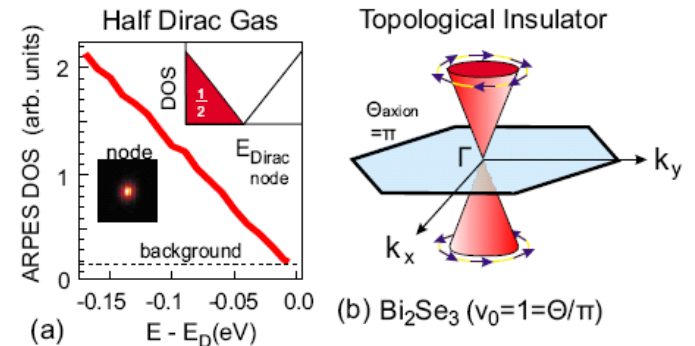
$$\hat{H}_{\text{SO}}^{\text{R}} = \frac{\alpha}{\hbar} (\hat{\sigma} \times \hat{\mathbf{p}})_z \equiv -\frac{g\mu_B}{2} \hat{\sigma} \cdot \mathbf{B}_R(\hat{\mathbf{p}})$$



# Crash Course on 3D Topological Insulators



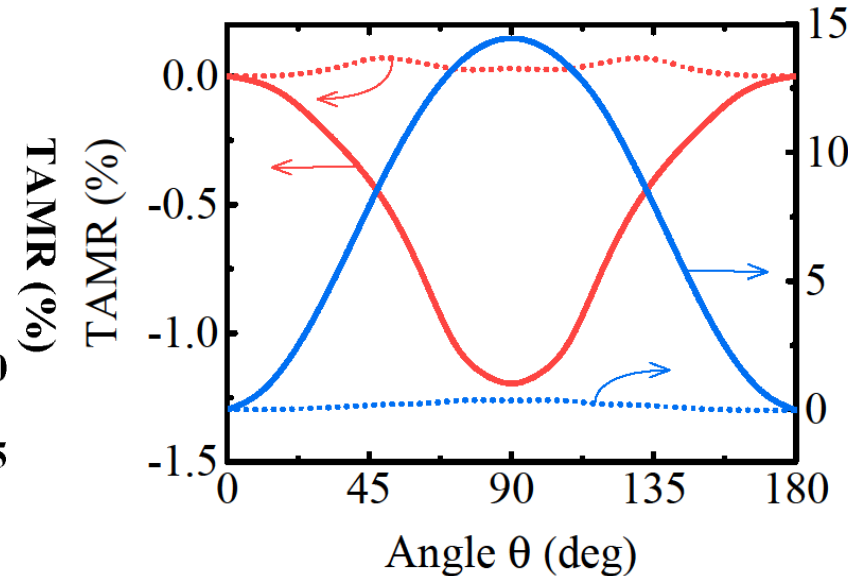
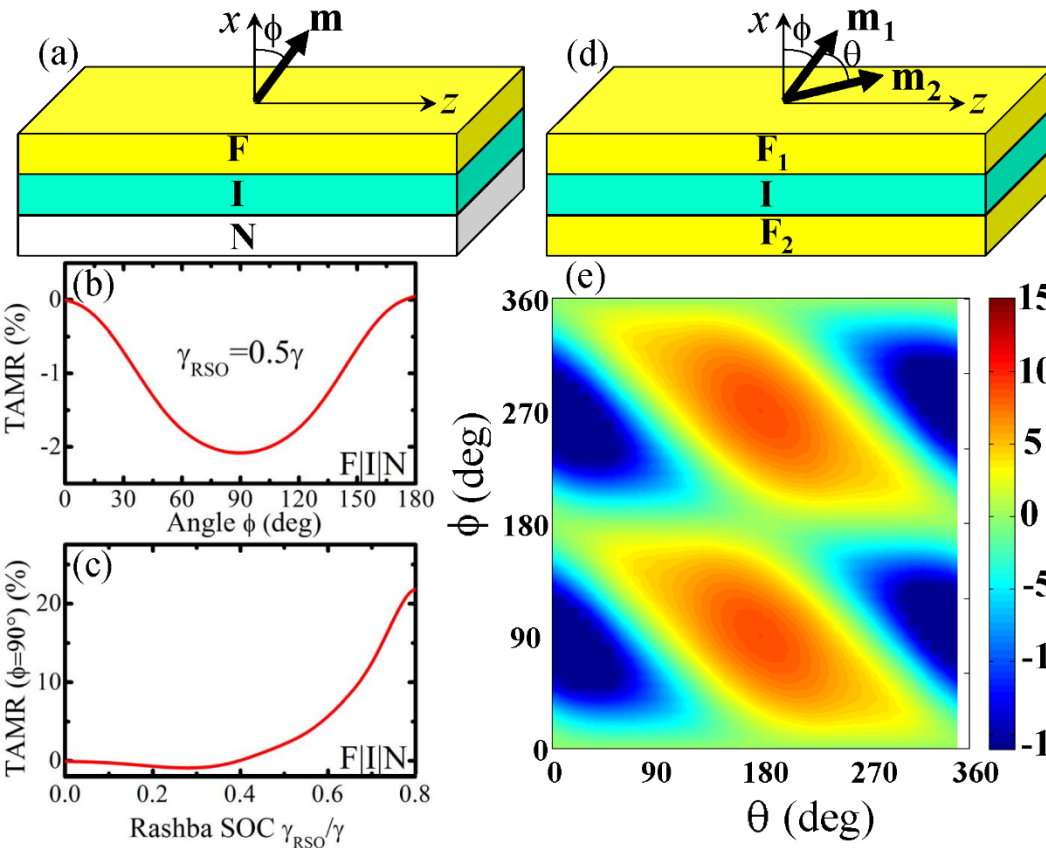
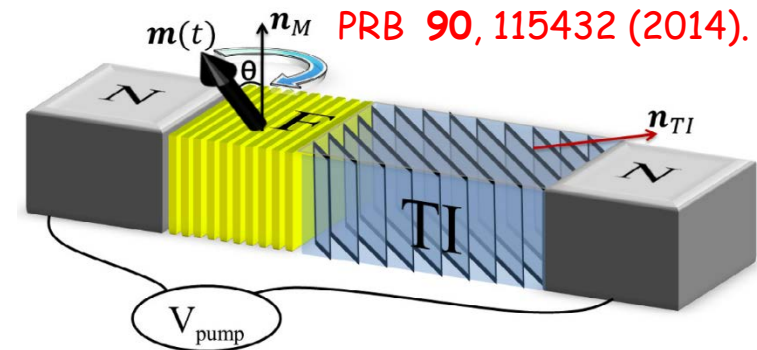
## Helical liquid on the surface of strong 3D TI



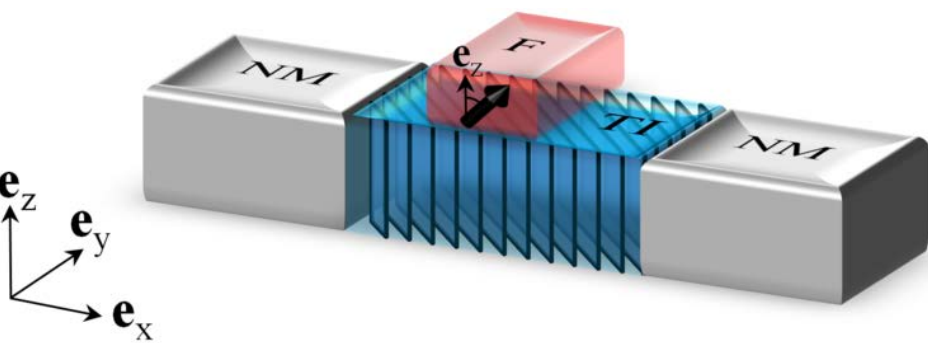
Why is this interesting: Protection by time-reversal invariance against scattering off non-magnetic impurities

# Detecting Interfacial SOC via Tunneling Anisotropic Magnetoresistance

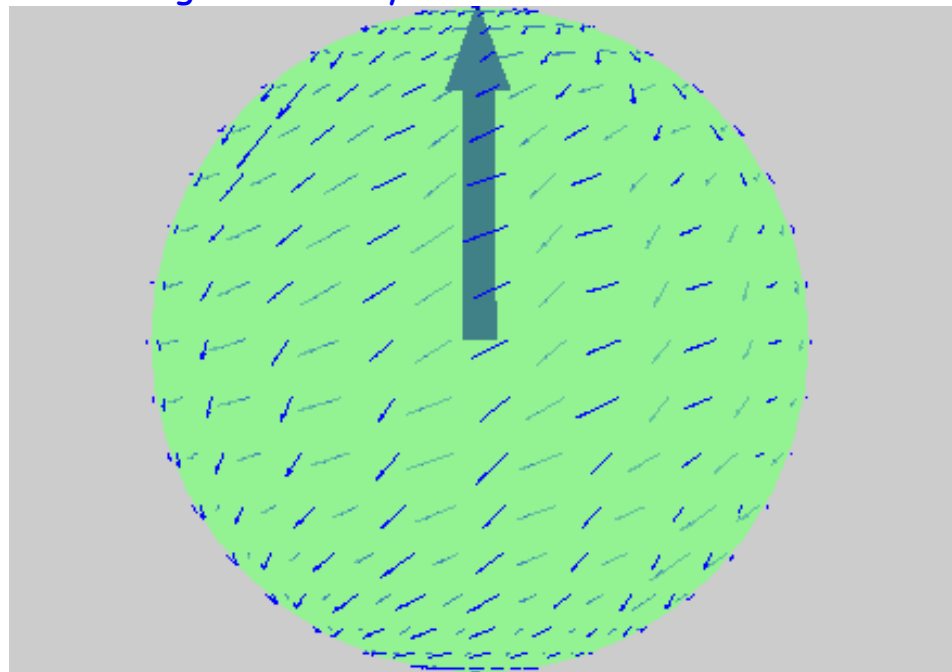
PRB **85**, 054406 (2012)



# STT in Lateral TI/FM Heterostructures



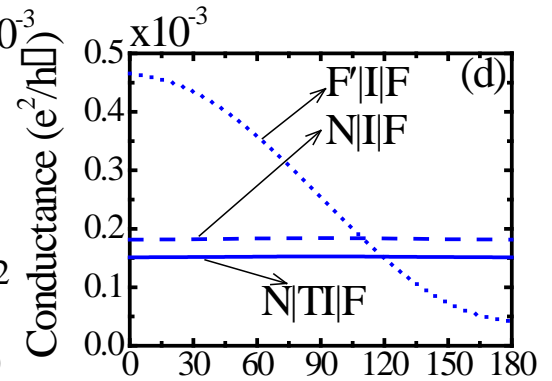
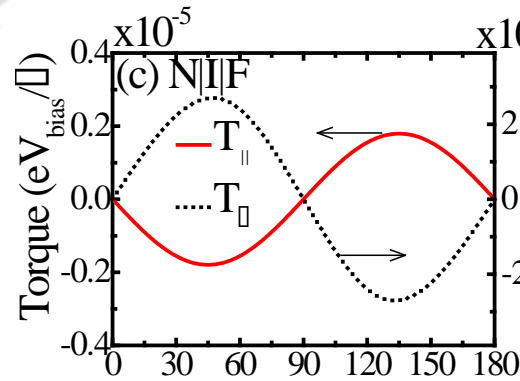
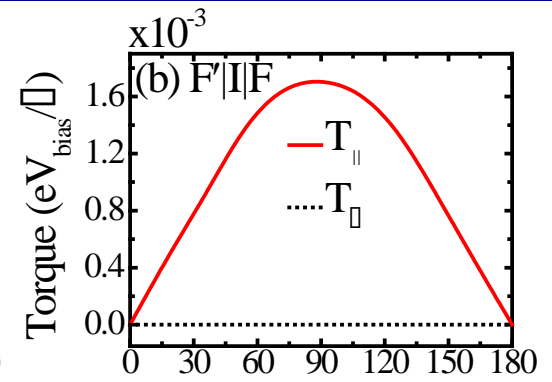
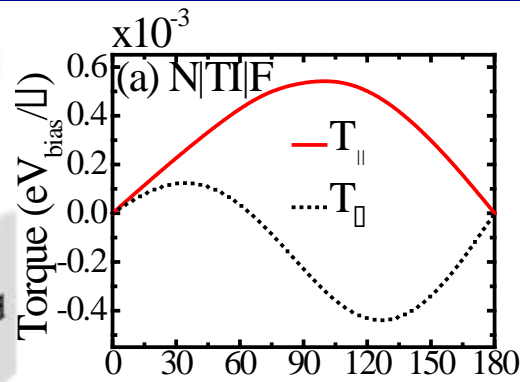
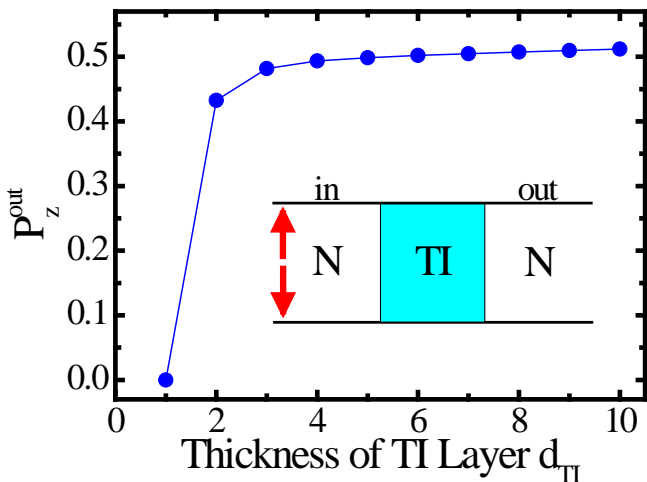
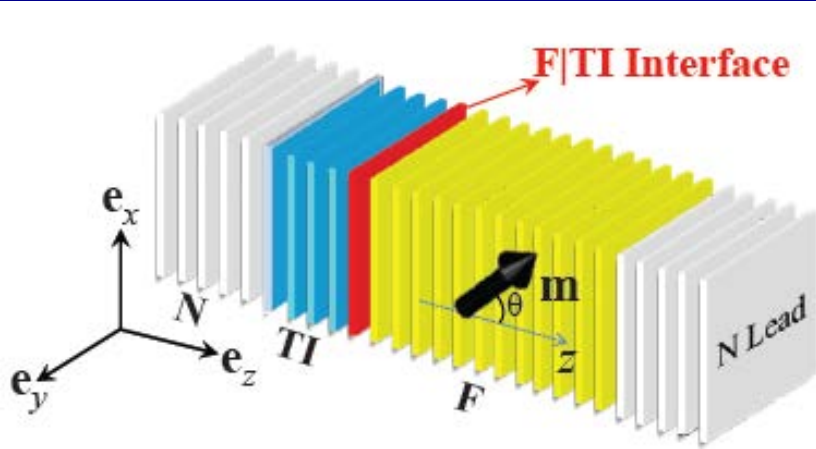
Solving LLG equation with torque field generated by the surface of 3D TI



unpublished



# STT in Vertical TI/FM Heterostructures



$$\mathbf{T} = \mathbf{T}_{\parallel} + \mathbf{T}_{\perp} = \tau_{\parallel} \mathbf{m} \times (\mathbf{m} \times \mathbf{e}_z) + \tau_{\perp} \mathbf{m} \times \mathbf{e}_z$$

PRB 71, 195328 (2005)

$$\hat{\rho}^{\text{out}} = \frac{1}{2}(1 + \mathbf{P}^{\text{out}} \cdot \boldsymbol{\sigma})$$

$$\hat{\rho}^{\text{out}} = \frac{e^2/h}{n_{\uparrow}(G^{\uparrow\uparrow} + G^{\downarrow\uparrow}) + n_{\downarrow}(G^{\uparrow\downarrow} + G^{\downarrow\downarrow})} \sum_{n',n=1}^M \begin{pmatrix} |t_{n'n,\uparrow\uparrow}|^2 + |t_{n'n,\uparrow\downarrow}|^2 & t_{n'n,\uparrow\uparrow}t_{n'n,\downarrow\uparrow}^* + t_{n'n,\uparrow\downarrow}t_{n'n,\downarrow\downarrow}^* \\ t_{n'n,\uparrow\uparrow}^*t_{n'n,\downarrow\uparrow} + t_{n'n,\uparrow\downarrow}^*t_{n'n,\downarrow\downarrow} & |t_{n'n,\downarrow\uparrow}|^2 + |t_{n'n,\downarrow\downarrow}|^2 \end{pmatrix}$$

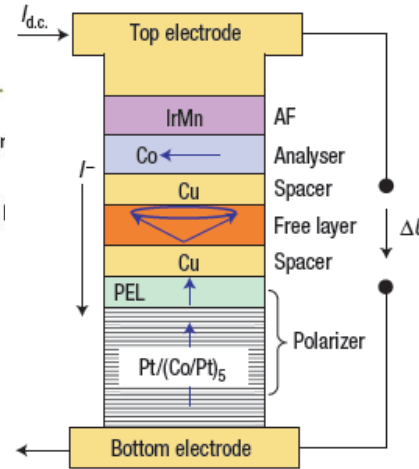
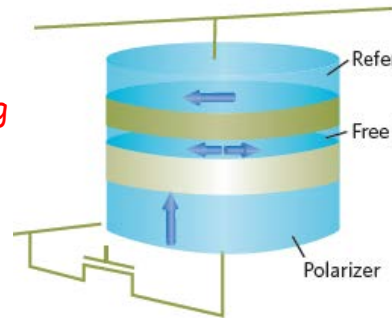
PRL 109, 166602 (2012)

# Applications: OSTT-MRAM and STT-nanooscillators using TI Capped MTJs

## MTJs for STT-MRAM applications

□ compromise between large current density (requiring low junction resistance to avoid damage) and readability (requiring large magnetoresistance)

□ optimization of the spin polarization across the junction, stabilization of the 'fixed' layer magnetization, and minimization of stray fields often result in complex stacking structures involving more than 10 different layers.



Supplemental Materials to  
PRL 109, 166602 (2012)

