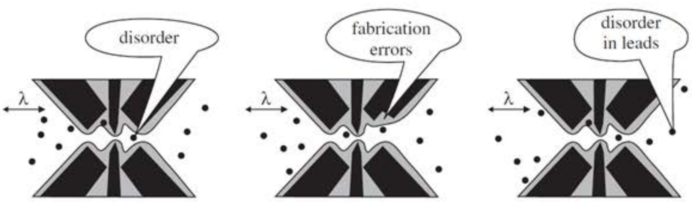
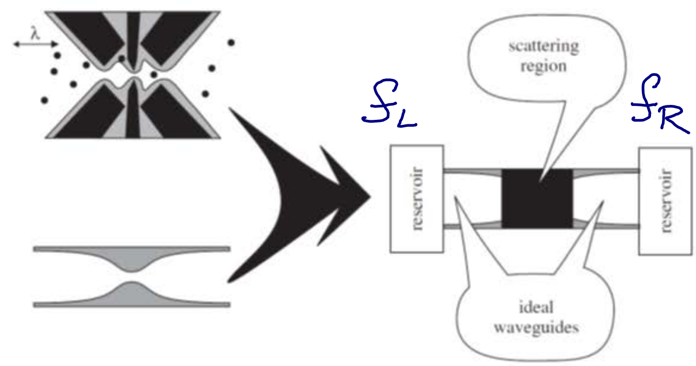
LECTURE 8: Landauer-Büttiker formula for two-terminal and multi-terminal quantum-coherent nanostructures

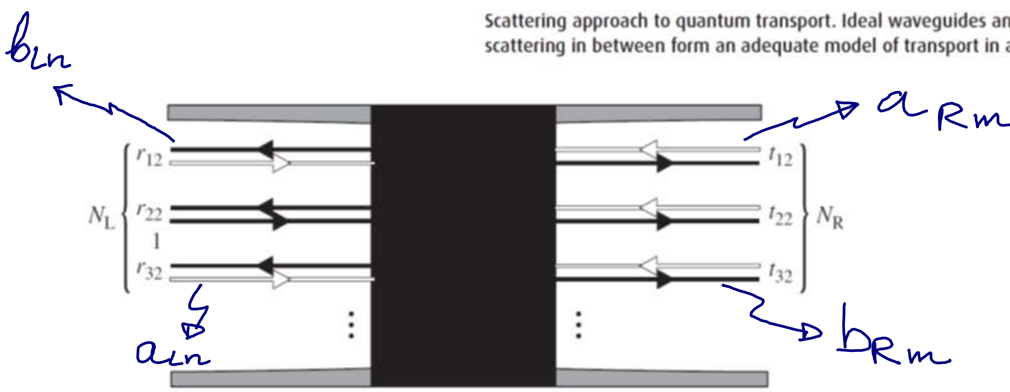
1° Scattering matrix



Nanostructures of an identical design are never identical.

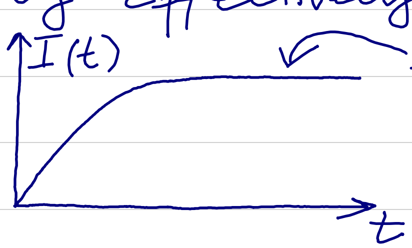


Scattering approach to quantum transport. Ideal waveguides and reservoirs from QPC plus scattering in between form an adequate model of transport in any nanostructure.



Structure of two-terminal scattering matrix. We show reflection and transmission amplitudes of the electron wave coming from the left in the second transport channel, $n' = 2$.

- ideal waveguides OR semi-infinite leads OR semi-infinite electrodes provide:
  - basis of quantum states ("tranverse propagating modes" or "conducting channels") in terms of which it is easy to express current at any cross section of the lead
  - continuous energy spectrum which plays important role in any nonequilibrium quantum system by effectively introducing dissipation and



steady-state current for dc bias voltage applied even even without explicit modeling of microscopic inelastic processes

→ the left (L) and right (R) leads do not have to have the same axis and the same cross section, so use:

$$x_L < 0,\ y_L,\ z_L \quad \text{and} \quad x_R > 0,\ y_R,\ z_R$$

→ single electron wavefunction in the leads are:

$$\Psi(x_L, y_L, z_L) = \sum_n \frac{1}{\sqrt{2\pi\hbar v_n}} \phi_n(y_L, z_L)\left[a_{Ln} e^{ik_x^{(n)} x_L} + b_{Ln} e^{-ik_x^{(n)} x_L}\right]$$

$$\Psi(x_R, y_R, z_R) = \sum_m \frac{1}{\sqrt{2\pi\hbar v_m}} \phi_m(y_R, z_R)\left[a_{Rm} e^{-ik_x^{(m)} x_R} + b_{Rm} e^{ik_x^{(m)} x_R}\right]$$

{ ϕ_n & ϕ_m are transverse wavefunctions in L & R lead, respectively, at the discrete energies E_n & E_m of the transverse motion

$$k_x^{(n)} = \sqrt{2m(E - E_n)/\hbar} \in \mathbb{R}$$

$$k_x^{(m)} = \sqrt{2m(E - E_m)/\hbar} \in \mathbb{R}$$

} transport is due to propagating, not evanescent, wavefunctions so that finite N_L and N_R channels are open at energy E

→ square root of velocity v_n in each channel ensures that current is expressed in terms of $a_{Ln}, b_{Ln}, a_{Rm}, b_{Rm}$ only

$$\vec{c}^{\,in} = \begin{pmatrix} a_{L1} \\ \vdots \\ a_{LN_L} \\ a_{R1} \\ \vdots \\ a_{RN_R} \end{pmatrix} \qquad \vec{c}^{\,out} = \begin{pmatrix} b_{L1} \\ \vdots \\ b_{LN_L} \\ b_{R1} \\ \vdots \\ b_{RN_R} \end{pmatrix}$$

scattering matrix

$$\vec{c}^{\,out} = \hat{S}\, \vec{c}^{\,in}$$

$$\Updownarrow$$

$$b_{\alpha\ell} = \sum_{\beta = L,R} \sum_{\ell' = n,m} S_{\alpha\ell, \beta\ell'}\, a_{\beta\ell'}$$

$$\hat{S} = \begin{pmatrix} \hat{S}_{LL} & \hat{S}_{LR} \\ \hat{S}_{RL} & \hat{S}_{RR} \end{pmatrix} = \begin{pmatrix} \hat{r} & \hat{t}' \\ \hat{t} & \hat{r}' \end{pmatrix}$$

⇒ $\hat{r}$ is $N_L \times N_L$ reflection matrix whose elements $r_{nn'}$ give probability $|r_{nn'}|^2$ for electron to impinge from channel n' in L lead and reflect into channel n also within L lead

⇒ $\hat{r}'$ is reflection matrix for channels in R lead

⇒ $\hat{t}$ is $N_R \times N_L$ transmission matrix where $|t_{mn}|^2$ gives probability for electron to impinge in channel n from the L lead and transmit through the scattering region into channel m in the R lead

2° Symmetries and properties of the scattering matrix

- UNITARITY ensures conservation of total probability

$$\hat{S}^+ \hat{S} = \hat{S}\hat{S}^+ = \hat{I} \Rightarrow \begin{cases} |\det S| = 1 \\ \sum_n |S_{nm}|^2 = \sum_m |S_{nm}|^2 = 1 \\ \hat{r}^+\hat{r} + \hat{t}^+\hat{t} = \hat{r}'^+\hat{r}' + \hat{t}'^+\hat{t}' = \hat{I} \\ \hat{r}\hat{r}^+ + \hat{t}'\hat{t}'^+ = \hat{r}'\hat{r}'^+ + \hat{t}\hat{t}^+ = \hat{I} \\ \hat{r}^+\hat{t}' + \hat{t}^+\hat{r}' = \hat{t}'^+\hat{r} + \hat{r}'^+\hat{t} = 0 \\ \hat{r}\hat{t}^+ + \hat{t}'\hat{r}'^+ = \hat{t}\hat{r}^+ + \hat{r}'\hat{t}'^+ = 0 \end{cases}$$

$$(\hat{S}^+\hat{S})_{nn} = \sum_{n'} |r_{nn'}|^2 + \sum_m |t_{mn}|^2 = 1$$

electron in channel n can be either transmitted or to any channel

TIME-REVERSAL or MOTION-REVERSAL

classical: $m\, d^2\vec{r}/dt^2 = \vec{F}$, if $t \mapsto -t$ both $\vec{r}(t)$ & $\vec{r}(-t)$ are solutions

quantum: $i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi$, if $t \mapsto -t$ both $\psi(\vec{r},t)$ and $\psi^*(\vec{r},-t)$ are solutions

$$\left[\frac{(i\hbar + e\vec{A})^2}{2m} + U(\vec{r})\right]\psi(\vec{r}) = E\,\psi(\vec{r}) \;\Big/ \underbrace{{}^{*}\;\vec{A} \to -\vec{A}}_{\text{motion reversal in QM}}$$

$$\underbrace{\left[\frac{(i\hbar + e\vec{A})^2}{2m} + U(\vec{r})\right]\psi^*(\vec{r}) = E\,\psi^*(\vec{r})}$$

$$[\psi^*(\vec{r})]_{-\vec{B}} = [\psi(\vec{r})]_{\vec{B}}$$

→ but taking complex conjugate makes incoming channel to become outgoing and vice versa:

$$\underbrace{\vec{c}^{\,out} = \hat{S}_{\vec{B}}\,\vec{c}^{\,in} \Rightarrow \vec{c}^{\,in} = \hat{S}^*_{-\vec{B}}\,\vec{c}^{\,out}}$$

$$\Downarrow$$

$$\hat{S}^{-1}_{\vec{B}} = \hat{S}^*_{-\vec{B}} \quad \text{together with} \quad \hat{S}^{-1} = \hat{S}^{\dagger}$$

gives $\hat{S}_{\vec{B}} = \hat{S}^{T}_{-\vec{B}}$

→ So, in the presence of magnetic field

$$t_{mn}(B) = t_{nm}(-B); \quad r_{nn'}(B) = r_{n'n}(-B)$$

$$t_{mn}(B) = t'_{nm}(-B); \quad r'_{mm'}(B) = r'_{m'm}(-B)$$

→ $\vec{B} = 0 \Rightarrow t_{mn} = t_{nm}$ or $\hat{t} = \hat{t}^{T}$ and $\hat{S} = \hat{S}^{T}$

3° Two-terminal Landauer–Büttiker formula

■ current at a cross section of L lead driven by dc bias voltage $eV_b = \mu_L - \mu_R$

$$I = 2_s e \sum_n \Bigg[\int_0^\infty \frac{dk_x}{2\pi} V_x(k_x) f_L(E)$$

$\to k_x > 0$ originate from L reservoir

$$k_x \mapsto -k_x \left\{ + \int_{-\infty}^0 \frac{dk_x}{2\pi} V_x(k_x) \sum_{n'} |r_{nn'}|^2 f_L(E) \right.$$

$\to k_x < 0$ originate from L reservoir but reflected

$$\left. + \int_{-\infty}^0 \frac{dk_x}{2\pi} V_x(k_x) \left(1 - \sum_{n'} |r_{nn'}|^2\right) f_R(E) \right\} \Bigg]$$

originate from right reservoir and transmitted into L lead

$$= 2_s e \sum_n \int_0^\infty \frac{dk_x}{2\pi} V_x(k_x) \underbrace{\left(1 - \sum_{n'} |r_{nn'}|^2\right)} [f_L(E) - f_R(E)]$$

$$V_x(k_x) = \frac{1}{\hbar} \frac{\partial E(k_x)}{\partial k_x} \qquad \text{by unitarity: } \sum_m |t_{mn}|^2 = (\hat{t}^+ \hat{t})_{nn}$$

$$I = \frac{2_s e}{2\pi} \int_0^\infty dE \, \mathrm{Tr}(\hat{t}^+ \hat{t}) [f_L(E) - f_R(E)]$$

$$\mathrm{Tr}(\hat{t}^+ \hat{t}) = \sum_n (\hat{t}^+ \hat{t})_{nn} = \sum_i T_i(E) = T(E)$$

$$\hat{t}^+ \hat{t} |T_i\rangle = T_i |T_i\rangle$$

T_i: transmission eigenvalues

$T(E)$: transmission function

■ Linear-response conductance

$$G = \lim_{V_b \to 0} I/V_b$$

$\to$ $eV_b \ll E_F$ in physics

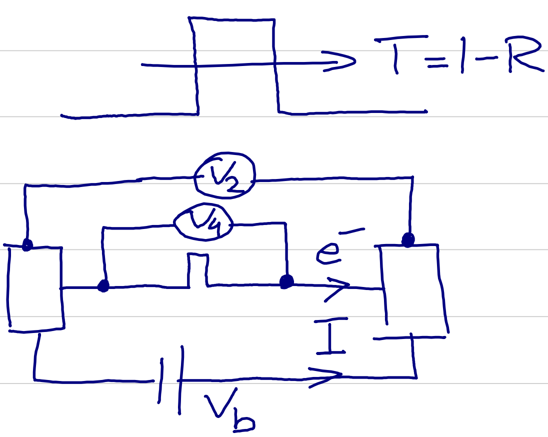
$$f_L(E) - f_R(E) \approx f(E) + \frac{\partial f}{\partial \mu}\mu_L - \left[f(E) + \frac{\partial f}{\partial \mu}\mu_R\right]$$

$$\approx \frac{\partial f}{\partial \mu}(\mu_L - \mu_R) = \left(-\frac{\partial f}{\partial E}\right)(\mu_L - \mu_R)$$

$$\boxed{G = \frac{I}{(\mu_L - \mu_R)/e} = \frac{2e^2}{h}\int dE\left(-\frac{\partial f}{\partial E}\right)\mathrm{Tr}\left(t^+ t\right)}$$

$$T_{emp} = 0 \Rightarrow f(E) = \Theta(E - E_F) \Rightarrow \left(-\frac{\partial f}{\partial E}\right) = \delta(E - E_F)$$

■ historically Landauer wrote this formula in 1D:



G_Q is conductance quantum

$$G_4 = \frac{2e^2}{h}\cdot\frac{T}{R}$$ which is the same as $G = \frac{2e^2}{h}\cdot T$ when $T \to 0$, but completely different in ballistic transport $T \to 1$

$$G_Q/G_2 = \frac{1}{T} = \frac{1-T}{T} + 1 = G_Q \cdot \frac{1}{G_4} + 1$$

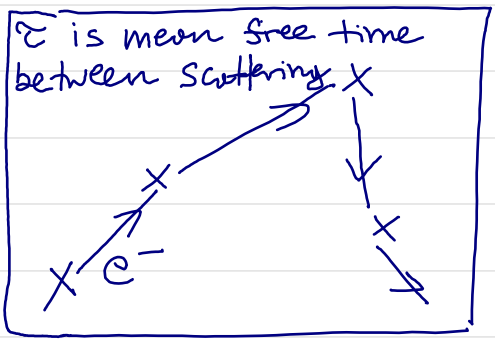
$\to$ four-terminal resistance V_4/I of wire itself

$$R_2 = R_4 + h/2e^2$$

$\to$ two-terminal resistance V_2/I

$\to$ contact resistance

4° Connection to semiclassical Drude formula



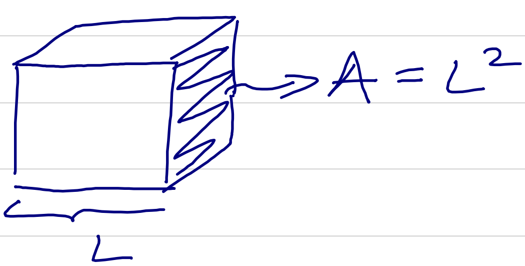
Bloch electrons at zero temperature have $\sigma \to \infty$, so impurities are needed for finite conductivity

$$m\, d\vec{v}/dt = e\vec{E} - \frac{m\vec{v}}{\tau} \Rightarrow m\frac{\vec{v}_{drift}}{\tau} = e\vec{E}$$

in steady-state

$$\text{mobility} = \frac{|\vec{v}_d|}{|\vec{E}|} = \frac{|e|\tau}{m}$$

$$\vec{j} = en\vec{v}_{drift} = \sigma \cdot \vec{E} \Rightarrow \sigma = \frac{ne^2\tau}{m} \quad \text{(Drude)}$$



$$\sigma = e^2 \underset{\uparrow\ \text{DOS}}{D(E_F)} \cdot \mathcal{D} \quad \text{(Einstein)}$$

$\mathcal{D} = \frac{1}{3} v_F \tau$ is diffusion constant in 3D

$$G = \sigma \cdot \frac{A}{L} = \frac{ne^2\tau}{m} \cdot \frac{A}{L}$$

$$n = 2_s \cdot \frac{4}{3} k_F^3 \pi \Big/ \left(\frac{2\pi}{L}\right)^3 \text{ for "spherical Fermi surface"}$$

or $\varepsilon = \hbar^2 k^2 / 2m$

$$G = \frac{2_s e^2}{h} \cdot \underbrace{\frac{k_F^2 A}{4\pi}}_{\text{number of channels } p \text{ with identical}} \cdot \frac{4}{3} \cdot \frac{\ell}{L} \Rightarrow T_i = \frac{4}{3}\frac{\ell}{L}$$

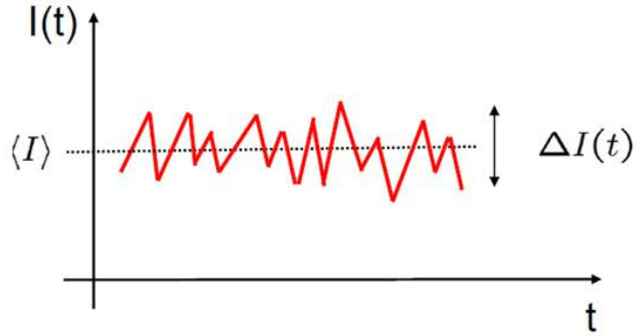
5° Distribution of transmission eigenvalues and its experimental probing by shot noise

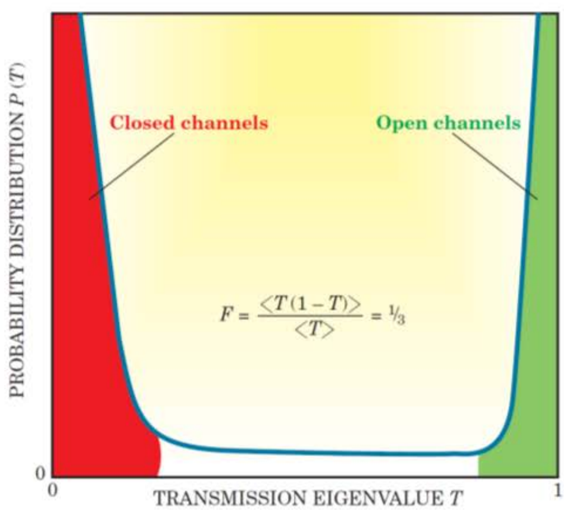
Quantum Shot Noise

Fluctuations in the flow of electrons can signal the transition from particlelike to wavelike behavior and signify the nature of charge transport in mesoscopic systems.

May 2003 Physics Today 37

Carlo Beenakker and Christian Schönenberger





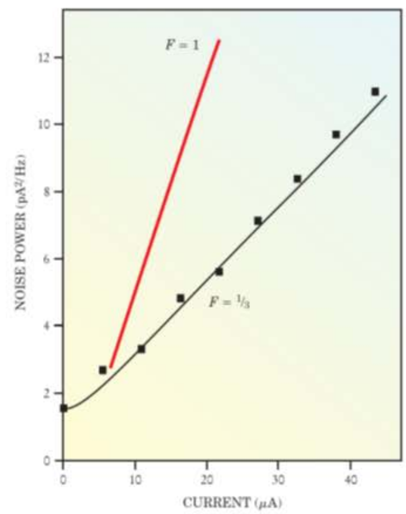


Figure 3. Sub-Poissonian shot noise in a disordered gold wire. At low currents, the black curve shows the noise saturate at the level set by the temperature of 0.3 K. Otherwise, the linear relation between noise power and current is the signature of shot noise. The slope is proportional to the Fano factor F, which measures the unit of transferred charge. Poissonian noise would have $F = 1$, drawn here as the red line. The experimental value $F = 1/3$ indicates the presence of strongly conducting transmission channels. (Adapted from M. Henny et al. ref. 11.)

$$P(T) = \overline{\sum_i \delta(T - T_i)}$$ ↳ average over disorder conf.

⇒ for diffusive wire Dorokhov distribution: $P_D(T) = \frac{G}{2G_Q} \frac{1}{T\sqrt{1-T}}$

$G = \int_0^1 T\, P(T)\, dT$ cannot differentiate between $P_{Drude}(T) = \text{const.}$ and $P_D(T)$

■ shot noise probes second moment of $P(T)$ → $\alpha = \beta$ is auto-correlation noise
→ $\alpha \neq \beta$ is cross-correlation noise

$$S_{pq}(t-t') = \frac{1}{2} \left\langle \delta\hat{I}_p(t)\, \delta\hat{I}_q(t') + \delta\hat{I}_q(t')\, \delta\hat{I}_p(t) \right\rangle$$

$$\delta\hat{I}_p(t) = \hat{I}_p(t) - \langle \hat{I}_p(t) \rangle$$

noise power spectral density

$$S_{pq}(\omega) = 2\int d(t-t')\, e^{-i\omega(t-t')} S_{pq}(t-t')$$

$$S_{LL} = S_{RR}(\omega=0, T_{exp}=0) = 2eF\langle I\rangle = \frac{4e^3 V_b}{h}\sum_i T_i(1-T_i)$$

$\hookrightarrow$ Fano factor

but it remains white at $\omega \neq 0$

$$F = \frac{\sum_i T_i(1-T_i)}{\sum_i T_i} = \frac{\int_0^1 T(1-T)P(T)\,dT}{\int_0^1 T P(T)\,dT}$$

$\Rightarrow$ for $P_D(T)$ one finds $F = 1/3$

$\Rightarrow$ for tunneling $T_i \to 0$ one finds $F = ?$

$\Rightarrow$ $F=1$ is Poisson limit as maximum value for noninteracting electrons transported as uncorrelated stochastic process where Pauli principle is inoperative

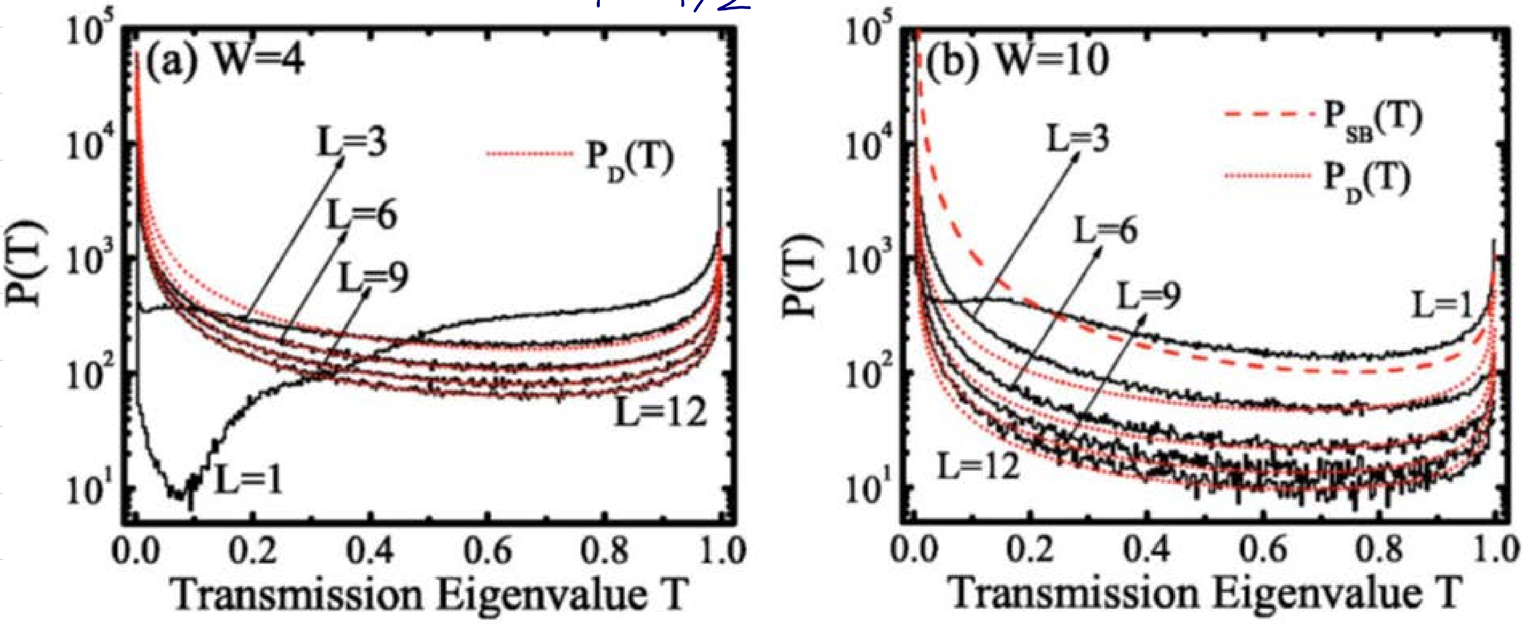
PHYSICAL REVIEW B 71, 045308 (2005)

Quantum transparency of Anderson insulator junctions: Statistics of transmission eigenvalues, shot noise, and proximity conductance

Branislav K. Nikolić and Ralitsa L. Dragomirova
Department of Physics and Astronomy, University of Delaware, Newark, Delaware 19716-2570, USA

$\rightarrow$ Schep-Bauer $P_{SB}(T) = \frac{G}{\pi G_Q}\frac{1}{T^{3/2}\sqrt{1-T}}$ for dirty interface

$\Leftrightarrow F = 1/2$

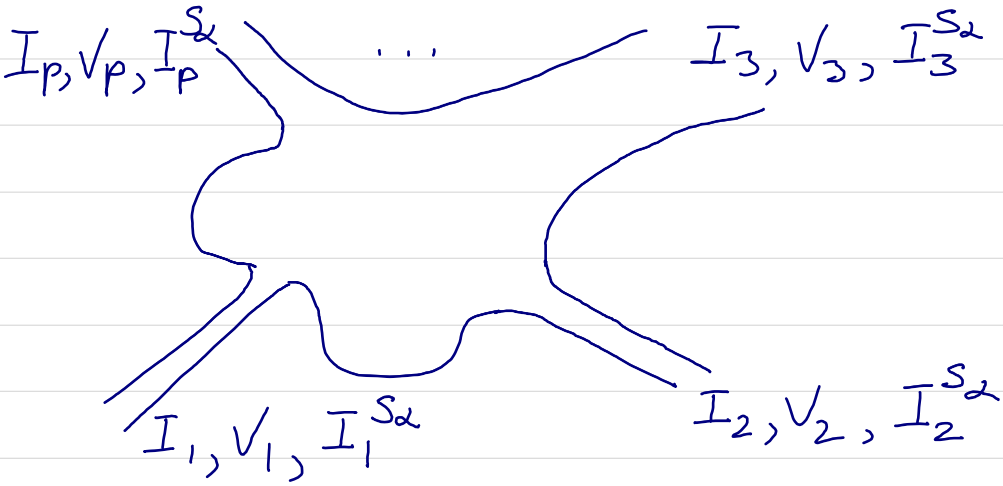


6° Multi-terminal Landauer-Büttiker formula

→ rewrite two-terminal current as:

$$I_1 = \frac{2e^2}{h} T_{2\leftarrow 1} (V_1 - V_2) = -I_2$$

→ generalize to more than two-terminals:



$$I_p = \frac{2e^2}{h} \sum_q \left(T_{q\leftarrow p} V_p - T_{p \leftarrow q} V_q \right)$$

$$= \sum_q \left(G_{qp} V_p - G_{pq} V_q \right)$$

$$T_{p\leftarrow q} = \mathrm{Tr}\left(\hat{t}^{\dagger}_{pq} \hat{t}_{pq} \right) \Rightarrow G_{pq} = \int dE \left(-\frac{\partial f}{\partial E} \right) T_{p\leftarrow q}(E)$$

→ gives probability for electron transmission from lead q to lead p

↳ conductance coefficient $[G_{qp}]_{+B} = [G_{pq}]_{-B}$

$$I_p = 0 \text{ for } V_p = \text{const.} \Rightarrow \begin{cases} \sum_q G_{qp} = \sum_q G_{pq} \\ I_p = \sum_q G_{pq} (V_p - V_q) \end{cases}$$

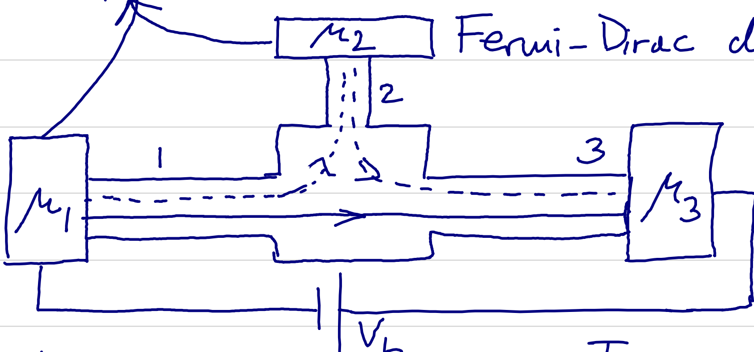
→ for spin currents insert Pauli matrices:

$$I_p^{S_\alpha} = \sum_q G_{pq}^{S_\alpha} (V_p - V_q)$$

$$G_{pq}^{S_\alpha} = \int dE \left(-\frac{\partial f}{\partial E}\right) \mathrm{Tr}\left(\hat{\sigma}_\alpha \hat{t}_{pq}^{+} \hat{t}_{pq}\right)$$

■ EXAMPLE: Büttiker voltage probe for dephasing

macroscopic reservoirs where electrons are equilibrated to Fermi-Dirac distribution with $\mu_i = E_F + eV_i$



$1 \to 2 \to 3$: electrons following this path lose memory of their phase by passing through reservoir 2

→ voltage probe means $I_2 \equiv 0$

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} G_{12}+G_{13} & -G_{12} & -G_{13} \\ -G_{21} & G_{21}+G_{23} & -G_{23} \\ -G_{31} & -G_{32} & G_{31}+G_{32} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

→ $V_3 = 0$ can be chosen arbitrarily as reference potential

$I_1 + I_2 + I_3 = 0$ is charge conservation or Kirchhoff rule

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} G_{12}+G_{13} & -G_{12} \\ -G_{21} & G_{21}+G_{23} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

inverse of

$I_1 = -I_3$

$I_2 = 0$ is Büttiker voltage probe send electron to reservoir, where the phase of its wavefunction is lost, and then injects it back into the central region while not drawing any current

$\Downarrow$

$$V_2 = \frac{T_{21}V_1 + T_{23}V_3}{T_{21}+T_{23}} = \frac{T_{21}V_1}{T_{21}+T_{23}} \quad \text{since we set } V_3 = 0$$

$$G = \frac{I_3}{V_1 - V_3} = \frac{2e^2}{h}\Big(\underbrace{T_{31}}_{\text{coherent}} + \underbrace{\frac{T_{32}T_{21}}{T_{21}+T_{23}}}_{\text{incoherent}}\Big)$$

→ fully COHERENT limit for dephasing probe decoupled:

$$T_{31} = T, \quad T_{21} = T_{23} = 0$$

$$G = \frac{2e^2}{h}T_{31} \Rightarrow R = \frac{h}{2e^2}\frac{1}{T_{31}}$$

↳ same as two-terminal conductance

→ fully INCOHERENT limit when direct transmission from 1 to 3 is supressed:

$$T_{3\leftarrow 1} = 0, \quad T_{2\leftarrow 1} = T_1, \quad T_{2\leftarrow 3} = T_2$$

$$G = \frac{2e^2}{h}\frac{T_1 T_2}{T_1 + T_2} \Rightarrow R = \frac{h}{2e^2}\left(\frac{1}{T_1} + \frac{1}{T_2}\right)$$

↳ classical addition of resistances in series

EXAMPLE: Two-terminal and four-terminal resistance in 2D topological insulator of quantum spin Hall type

Experiment:

Nonlocal Transport in the Quantum Spin Hall State

Andreas Roth,[1] Christoph Brüne,[1] Hartmut Buhmann,[1] Laurens W. Molenkamp,[1*] Joseph Maciejko,[2,3] Xiao-Liang Qi,[2,3] Shou-Cheng Zhang[2,3]

294 17 JULY 2009 VOL 325 SCIENCE

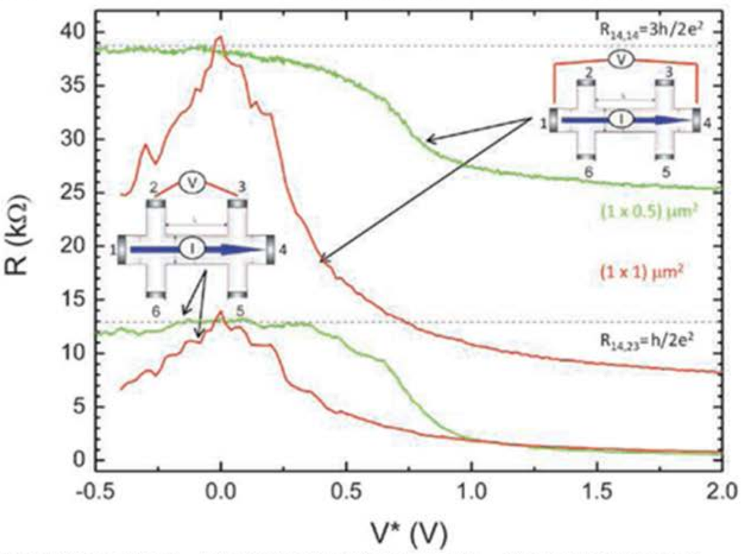
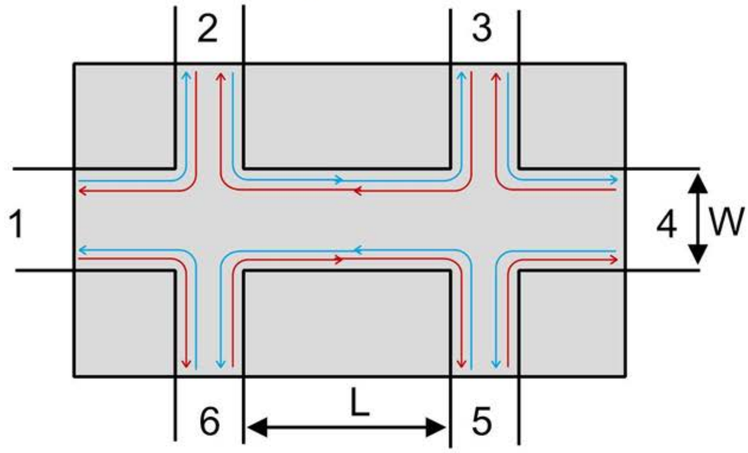


Fig. 1. Two-terminal ($R_{14,14}$) (top two traces) and four-terminal ($R_{14,23}$) (bottom traces) resistance (normalized) gate voltage for the Hall bar devices D1 and D2 with dimensions (length × width) as indi The dotted blue lines indicate the resistance values expected from the Landauer-Büttiker approach.

Theory & Computation:



$$T_{i\leftarrow i+1} = T_{i+1\leftarrow i} = 1 \; ; \; i = 1, \dots, 6$$

transmission is perfect due to spin-polarized and chiral (or "helical") edge edge states of quantum spin Hall type of 2D topological insulator

NOTE: requires to assume absence of magnetic impurities which can flip spin and allow backscattering with $T_{i\leftarrow i+1} < 1$

$$I_{4\leftarrow 1}\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{pmatrix} = \frac{e^2}{h} \underbrace{\begin{pmatrix} -2 & 1 & 0 & 0 & 0 & 1 \\ 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 0 \\ 1 & 0 & 0 & 0 & 1 & -2 \end{pmatrix}}_{\text{determinant}=0} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{pmatrix}$$

determinant = 0 but we can remove redundancy by $V_4 = 0$

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \end{pmatrix} = I_{4\leftarrow 1} \frac{h}{e^2} \begin{pmatrix} -2 & 1 & 0 & 0 & 1 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & -2 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \end{pmatrix} = I_{4\leftarrow 1} \frac{h}{e^2} \begin{pmatrix} -3/2 \\ -1 \\ -1/2 \\ -1/2 \\ -1 \end{pmatrix}$$

$$V_4 - V_1 = \frac{3h}{2e^2} I_{4\leftarrow 1} \Rightarrow \boxed{R_{14,14} = \frac{V_1 - V_4}{I_{4\leftarrow 1}} = \frac{3h}{2e^2}}$$

$$V_3 - V_2 = \frac{h}{2e^2} I_{4\leftarrow 1} \Rightarrow \boxed{R_{14,23} = \frac{V_3 - V_2}{I_{4\leftarrow 1}} = \frac{h}{2e^2}}$$

compare theoretical prediction based on topological edge states with experimental measurements

EXAMPLE: Spin Hall angle and nonlocal resistance

Experiment:

ARTICLE

Received 17 Apr 2014 | Accepted 18 Jul 2014 | Published 1 Sep 2014 DOI: 10.1038/ncomms5748

Giant spin Hall effect in graphene grown by chemical vapour deposition

Jayakumar Balakrishnan[1,2,*,†], Gavin Kok Wai Koon[1,2,3,*], Ahmet Avsar[1,2], Yuda Ho[1,3], Jong Hak Lee[1,2], Manu Jaiswal[1,2,†], Seung-Jae Baeck[4], Jong-Hyun Ahn[4], Aires Ferreira[1,2,5], Miguel A. Cazalilla[2,6], Antonio H. Castro Neto[1,2] & Barbaros Özyilmaz[1,2,3]

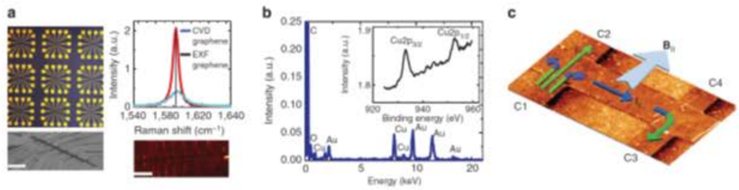


Figure 1 | Device characterization. (**a**) Optical image of 3 × 3 array of devices on Si/SiO$_2$ substrate together with the Raman and SEM image of the active area of a typical spin Hall device. Scale bar, 5 μm. (**b**) EDX spectrum of CVDG. The samples for EDX measurements are prepared on a standard transmission electron microscopy (TEM) gold grids and are hence suspended graphene samples. The size of each grid is 7 × 7 μm. The additional Au peaks in the EDX spectrum is due to the presence of Au TEM grids on which the graphene samples are prepared for analysis. Inset: XPS data on a CVD graphene sample showing the Cu 2p peaks. (**c**) AFM three-dimensional surface topography of a typical spin Hall device with details of actual measurement configurations.

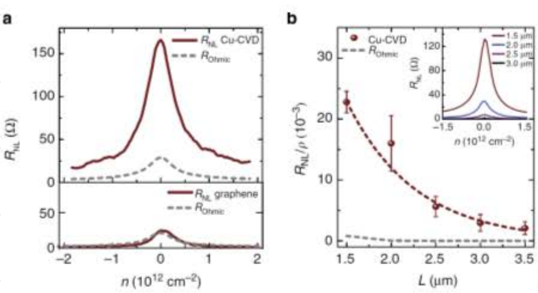


Table 1 | Graphene decorated with metallic adatoms.

Adatom	Mobility (cm^{2}V^{-1}s^{-1})	λ_s (μm)	γ	Δ (meV)
Cu-CVD	11,000	1.9	0.17	14.4
Cu-EPG	9,000	1.1	0.27	17.4
Au-EPG	15,000	2.0	0.15	18.0

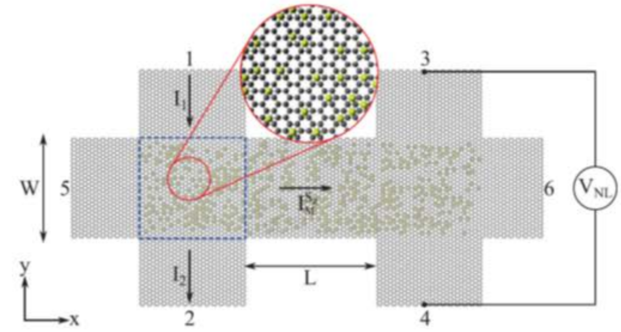
CVD, chemical vapour deposition; EPG, exfoliated pristine graphene.
The extracted values for Δ assume predominant intrinsic SOC (see main text).

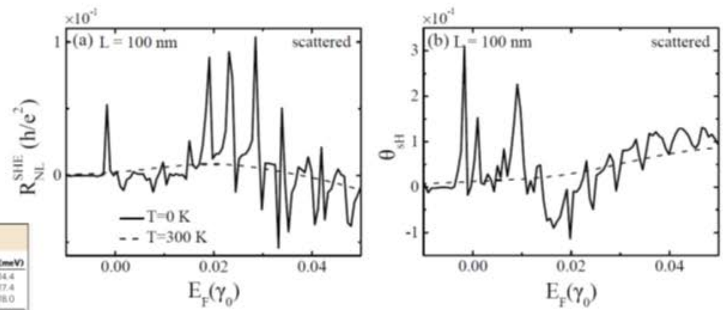
Theory & Computation:

PRL 117, 176602 (2016) PHYSICAL REVIEW LETTERS week ending 21 OCTOBER 2016

Spin Hall Effect and Origins of Nonlocal Resistance in Adatom-Decorated Graphene

D. Van Tuan,[1,2] J. M. Marmolejo-Tejada,[3,4] X. Waintal,[5] B. K. Nikolić,[3,*] S. O. Valenzuela,[1,6] and S. Roche[1,6,7]





⟹ spin Hall angle θ_{SH}:

$\underbrace{I_5 = I_6 = 0}$ are conditions to have pure transverse spin Hall current in response to longitudinal injected charge current $I_1 \neq 0 \neq I_2$

$\Downarrow$

find V_5 & V_6 and $I_5^{S_z}$ ⟶ compute V_3 and V_4 for these conditions

$\Downarrow$

$$\boxed{\theta_{SH} = \frac{I_5^{S_z}}{I_1}}$$

⟹ nonlocal resistance: $R_{NL} = \dfrac{V_{NL}}{I_1} = \dfrac{V_3 - V_4}{I_1}$