A Letters Journal Exploring the Frontiers of Physics

February 2007

EPL, **77** (2007) 47004
doi: 10.1209/0295-5075/77/47004

www.epljournal.org

Extrinsically vs. intrinsically driven spin Hall effect in disordered mesoscopic multiterminal bars

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received 7 November 2006; accepted 21 December 2006
published online 12 February 2007

PACS 72.25.Dc – Spin polarized transport in semiconductors
PACS 73.63.-b – Electronic transport in nanoscale materials and structures

Abstract – We show that pure spin Hall current, flowing out of a four-terminal two-dimensional electron gas (2DEG) within inversion asymmetric heterostructure, contains contributions from both the extrinsic mechanisms (spin-orbit-dependent scattering off impurities) and the intrinsic ones (due to the Rashba coupling). The extrinsic contribution vanishes in the weakly and strongly disordered limits, and the intrinsic one dominates in the quasiballistic limit. However, in the crossover transport regime the spin Hall conductance is not simply reducible to either of the two mechanisms, which can be relevant for interpretation of experiments on dirty 2DEGs (Sih V. *et al.*, *Nature Phys.*, **1** (2005) 31). We also predict sample-to-sample mesoscopic fluctuations, and even sign change, of both the spin Hall conductance and the applied voltages to transverse terminals (ensuring that spin Hall current outflowing through them is pure with no accompanied net charge current) in phase-coherent bars.

Introduction. – Recent experimental discovery of the spin Hall effect in three-dimensional n-type semiconductors [1] and two-dimensional hole [2] or electron gases [3] provides deep insight into the relativistic effects in solids which manifest through the spin-orbit (SO) couplings [4]. The flow of conventional unpolarized charge current in the longitudinal direction through systems governed by SO interactions can lead to spin flux in the transverse direction which deposits non-equilibrium spin Hall accumulation on the lateral sample edges, as observed in experiments [1–3]. Moreover, anticipated experiments should demonstrate flow of spin into the transverse electrodes attached at those edges [5], thereby offering a semiconductor analog of the Stern-Gerlach device as a source of spin currents or flying spin qubits where spatial separation of spin-↑ and spin-↓ electrons does not require any external magnetic fields.

However, the effect observed in electron systems [1,3] is rather small, and its magnitude cannot be tuned easily since it is determined by material parameters which govern the SO-dependent scattering off impurities [6–8] deflecting a beam of spin-↑ (spin-↓) electrons predominantly to the right (left). In contrast to such extrinsically (*i.e.*, impurity) induced spin Hall effect, recent theories have argued for several orders of magnitude larger spin Hall currents due to the intrinsic SO couplings capable of spin-splitting the energy bands and inducing transverse SO forces in bulk semiconductors in the *clean* limit [9,10], as well as in *ballistic* mesoscopic multiterminal nanostructures made of such materials [5,11]. Such mechanisms were invoked to explain [12] observation of two orders of magnitude larger spin Hall accumulation in 2D hole gases [2], whose magnitude can still be affected by the effects of the disorder [13]. Moreover, in infinite 2DEGs with linear in momentum SO coupling any non-vanishing spin-independent disorder completely suppresses the spin Hall effect [14,15], while in open finite-size 2DEG structures attached to external current and voltage probes increasing disorder can only gradually diminish its signatures [11,15,16].

Despite these advances, the *intrinsic vs. extrinsic* debate on the origin of the detected signatures of the spin Hall effect persists [17,18], closely mimicking several decades old discourse [19] on the Karplus-Luttinger *intrinsic* (due to anomalous velocity, interpreted in modern language via the Berry phase associated with electronic band structure) *vs. extrinsic* (*i.e.*, skew-scattering + side-jump) explanations of the anomalous Hall effect in ferromagnetic systems. Even though much of this controversy might be resolved through a unified theoretical treatment of different mechanisms [19], surprisingly enough, *no theory* has been constructed to describe spin Hall currents in the transition from quasiballistic to dirty transport regime in *multiterminal bars* employed in experiments where both extrinsic and intrinsic contributions could play an equally important role [20]. Also, quantum corrections to the semiclassical description [7,8] of the extrinsic spin Hall effect, such as the mesoscopic fluctuations and localization effects due to the interplay of randomness and quantum coherence, can be important for interpreting experiments on meso- and nano-scale samples at low temperatures.

The demand for such theory is further provoked by a recent spin Hall experiment on dirty two-terminal 2DEG within AlGaAs quantum well which accommodates both the SO scattering off impurities and a small Rashba SO coupling due to weak structural inversion asymmetry. The latter could give rise to an intrinsic contribution of a similar magnitude as the extrinsic one for not too small ratio [11,16] $\Delta_{\rm SO}\tau/\hbar$ ($\Delta_{\rm SO}$ is the spin-splitting of quasiparticle energies and $\hbar/\tau$ is the disorder-induced broadening of energy levels due to finite transport scattering time τ) —the parameters measured in ref. [3] place its Hall bar into the regime $\Delta_{\rm SO}\tau/\hbar \sim 10^{-2}$.

Here we construct a mesoscopic transport theory (based on the spin-resolved scattering approach [11,16]) to spin Hall fluxes in multiterminal bars driven by a generic Pauli Hamiltonian

$$\hat{H} = \frac{\hat{p}_x^2+\hat{p}_y^2}{2m^*} + V_{\rm confine}(y) + V_{\rm disorder}(x,y) - \frac{g\mu_B}{2}\hat{\boldsymbol{\sigma}}\cdot\mathbf{B}(\mathbf{p}) + \lambda\,(\hat{\boldsymbol{\sigma}}\times\hat{\boldsymbol{p}})\cdot\nabla V_{\rm disorder}(x,y), \quad (1)$$

which encompasses both the intrinsic (fourth term) and the extrinsic (fifth term) SO coupling-induced effects. Within this formalism, whose technical aspects are elaborated below, we obtain in fig. 1 the spin Hall *conductance* in the transverse ideal (*i.e.*, spin and charge interaction free) leads of a four-terminal quantum-coherent Hall bar containing such 2DEG. The spin conductance is defined as $G_{sH} = I_2^{S_z}/(V_1 - V_4)$ for *pure* ($I_2 = I_2^{\uparrow} + I_2^{\downarrow} = 0$) total spin current $I_2^{S_z} = \frac{\hbar}{2e}(I_2^{\uparrow} - I_2^{\downarrow})$ of z-polarized spins driven through transverse lead 2 by the bias voltage $V_1 - V_4$ (see inset of fig. 1 for labeling the leads of the bar where the basic transport quantity is the total spin-resolved charge current of spin-σ electrons I_p^{σ} in lead p).

The intrinsic SO coupling term has the general Zeeman form $-g\mu_B\hat{\boldsymbol{\sigma}}\cdot\mathbf{B}(\mathbf{p})/2$, but its effective magnetic field $\mathbf{B}(\mathbf{p})$ is momentum dependent and does not break the time-reversal invariance. In the case of the Rashba SO coupling, which arises due to structural inversion asymmetry (of the 2DEG confining electric potential along the z-axis and differing band discontinuities at the heterostructure quantum well interface [4]), the "internal" magnetic field is $\mathbf{B}(\mathbf{p}) = -(2\alpha/g\mu_B)(\hat{\mathbf{p}}\times\hat{z})$, where $\hat{z}$ is the unit vector orthogonal to 2DEG. The corresponding spin-splitting of the energy bands at the Fermi level ($\hbar k_F$ is the Fermi momentum) is given by $\Delta_{\rm SO} = g\mu_B|\mathbf{B}(\mathbf{p})| = 2\alpha k_F$.

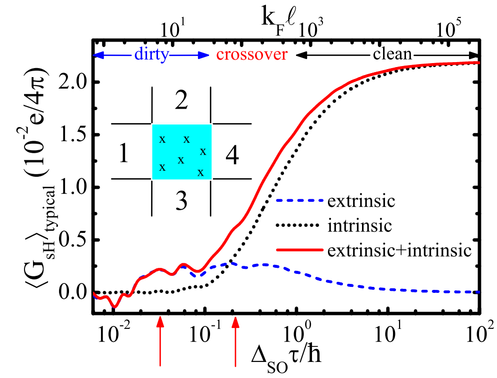

Fig. 1: (Colour on-line) The disorder-averaged *typical* spin Hall conductance of a four-terminal phase-coherent 2DEG nanostructure of the size $100a\times 100a$ ($a \simeq 2$ nm), which is governed by both the SO scattering off impurities (setting the transport scattering time τ) of strength $\lambda_{\rm SO}$ and the intrinsic Rashba SO coupling responsible for the energy spin-splitting $\Delta_{\rm SO} = 4at_{\rm SO}k_F$. Following measured parameters of 2DEG in ref. [3], we use $\lambda_{\rm SO} = 0.005$, $t_{\rm SO} = 0$ for the "extrinsic"; $\lambda_{\rm SO} = 0$, $t_{\rm SO} = 0.003t_{\rm o}$, for the "intrinsic"; and $\lambda_{\rm SO} = 0.005$, $t_{\rm SO} = 0.003t_{\rm o}$ for the "extrinsic + intrinsic" spin Hall conductance $G_{sH} = I_2^{S_z}/(V_1 - V_4)$ quantifying the pure spin current $I_2^{S_z}$. The leftmost red arrow *confirms* that observed signatures of the spin Hall effect in ref. [3] are mostly due to extrinsic mechanisms.

The Thomas term $\lambda\,(\hat{\boldsymbol{\sigma}}\times\hat{\boldsymbol{p}})\cdot\nabla V_{\rm dis}(\mathbf{r})$ is a relativistic correction to the Pauli equation for the spin-$\frac{1}{2}$ particle, where $\lambda/\hbar = -\hbar^2/4m_0^2c^2 \approx -3.7\times 10^{-6}$ Å^2 (c is the velocity of light) for an electron with mass m_0 in vacuum. Such minuscule value of λ would lead to a hardly observable spin Hall effect. However, in solids it can be enormously renormalized by the band structure effects due to strong crystal potential. For example, $\lambda/\hbar = 5.3$ Å^2 in GaAs [4] generates the extrinsic spin Hall conductivity $\sigma_{sH}^{\rm extr} = j_y^z/E_x$, whose recently computed value [7,8] accounts well for the magnitude of the observed spin Hall accumulation in the dirty 3D electron systems of ref. [1]. In the inversion symmetric structures $\mathbf{B}(\mathbf{p}) \equiv 0$, so that the pure spin current j_y^z (carrying z-axis polarized spins in the y-direction) is driven solely by the Thomas term with $\lambda \neq 0$ in the presence of the longitudinal external electric field E_x.

The extrinsic spin Hall *conductivity* $\sigma_{sH}^{\rm extr} = \sigma_{sH}^{\rm SS} + \sigma_{sH}^{\rm SJ}$, which is the sum [7,8] of the skew-scattering $\sigma_{sH}^{\rm SS}$ and the side-jump $\sigma_{sH}^{\rm SJ}$ contributions of opposite sign, is a singular function as the disorder strength is decreased to zero since $\sigma_{sH}^{\rm SS} \sim \tau$, while $\sigma_{sH}^{\rm SJ}$ is τ-independent [7,8]. Therefore, in the clean limit this divergence is expected to be cut off at $\tau \sim \hbar/\Delta_{\rm SO}$, where $\Delta_{\rm SO}$ is due to the k^3 Dresselhaus term in the absence of Rashba coupling [7]. In contrast to this unphysical behavior of bulk transport quantities, fig. 1 shows that the extrinsic spin Hall *conductance* $G_{sH}^{\rm extr} = G_{sH}(\lambda\neq 0, \alpha = 0)$ of the four-probe mesoscopic bar goes to zero $G_{sH}^{\rm extr} \to 0$ in the "clean" regime (weak disorder or strong Rashba coupling) $\Delta_{\rm SO} \gg \hbar/\tau$, as well as in the very ($k_F\ell \sim 1$) "dirty" regime (strong disorder or weak

Rashba coupling) $\Delta_{\rm SO} \ll \hbar/\tau$ where semiclassical theories break down.

Scattering approach to extrinsic + intrinsic total spin Hall currents in terminals of mesoscopic bars. – Theoretical analysis of the spin Hall effect in bulk systems yields σ_{sH} by computing (via the Kubo formula [8–10,14] or kinetic equations [7]) the pure spin current density j_y^z, as the expectation value of some plausibly introduced spin current operator $\hat{j}_y^z$ [18], in linear response to external electric field E_x penetrating an infinite semiconductor. This procedure is well defined in inversion symmetric systems (such as $\lambda \neq 0$, $\alpha = 0$) where the SO contribution to the impurity potential decays fast and spin current is a conserved quantity [7]. However, the non-conservation of spin in spin-split systems with $\mathbf{B}(\mathbf{p}) \neq 0$ leads to ambiguity in the definition of spin current, making it difficult to connect σ_{sH} and spin accumulation [18]. That is, the spin density relaxes not only due to the spin flux but also due to spin precession, thereby requiring a careful treatment of boundary effects on the observed spin accumulation [21].

To generate spin Hall phenomenology in multiterminal bars that is analogous to the bulk theoretical studies, we have to employ at least four external leads where passing unpolarized charge current through the longitudinal electrodes generates spin current in the transverse electrodes [5,11,15]. The charge currents in the leads are described by the same multiprobe Landauer-Büttiker scattering formulas employed for quantum Hall bars [22], $I_p = \sum_q G_{pq}(V_p - V_q)$. They relate the charge current I_p in probe p to voltages V_q in all probes attached to the sample via conductance coefficients G_{pq} that intrinsically take into account all interfaces and boundaries. To ensure that the transverse spin Hall current is *pure*, one has to apply the transverse voltages V_2 and V_3 obtained from these equations by setting $I_2 = I_3 = 0$. The extension of the scattering theory of quantum transport to *total* [5] spin currents in the terminals $I_p^S = \frac{\hbar}{2e}(I_p^{\uparrow} - I_p^{\downarrow})$, which are always genuine *non-equilibrium* response (in contrast to spin current density which is non-zero even in equilibrium [5]) and *conserved* in the ideal $[\mathbf{B}(\mathbf{p}) \equiv 0]$ leads, gives for the multiprobe spin current formulas [11]

$$I_p^S = \frac{\hbar}{2e} \sum_{q \neq p} (G_{qp}^{\rm out} V_p - G_{pq}^{\rm in} V_q). \tag{2}$$

Here $G_{pq}^{\rm in} = G_{pq}^{\uparrow\uparrow} + G_{pq}^{\uparrow\downarrow} - G_{pq}^{\downarrow\uparrow} - G_{pq}^{\downarrow\downarrow}$, $G_{qp}^{\rm out} = G_{qp}^{\uparrow\uparrow} + G_{qp}^{\downarrow\uparrow} - G_{qp}^{\uparrow\downarrow} - G_{qp}^{\downarrow\downarrow}$, and usual charge conductances are $G_{pq} = G_{pq}^{\uparrow\uparrow} + G_{pq}^{\uparrow\downarrow} + G_{pq}^{\downarrow\uparrow} + G_{pq}^{\downarrow\downarrow}$. Since the outflowing spin current I_p^S is computed in the ideal electrode with no SO coupling [5], this formalism also evades the issue of incorrect velocity operator in the bulk (whose usage results in two times smaller $\sigma_{sH}^{\rm SJ}$ [7]) obtained directly from the Hamiltonian eq. (4) while neglecting the anomalous SO contribution that introduces a gauge covariant position operator and the non-commutativity of coordinates [23].

The spin-resolved conductances are computed from the transmission matrices $\mathbf{t}^{pq}$ between the leads p and q through the Landauer-type formula $G_{pq}^{\sigma\sigma'} = \frac{e^2}{h} \sum_{i,j=1}^{M} |\mathbf{t}_{ij,\sigma\sigma'}^{pq}|^2$, where $\sum_{i,j=1}^{M} |\mathbf{t}_{ij,\sigma\sigma'}^{pq}|^2$ is the probability for spin-σ' electron incident in lead q to be transmitted to lead p as spin-σ electron and $i, j = 1, \ldots, M$ label the transverse propagating modes in the leads. Solving eq. (2) for I_2^S in the top transverse lead of the Hall bar in fig. 1 yields

$$G_{sH} = \frac{\hbar}{2e} \left[\frac{G_{24}^{\rm in} - G_{21}^{\rm in}}{2} + \left(G_{12}^{\rm out} + G_{32}^{\rm out} + G_{42}^{\rm out}\right) \frac{V_2}{V} - G_{23}^{\rm in} \frac{V_3}{V} \right], \tag{3}$$

assuming $V_1 = V/2 = -V_4$ for the applied bias voltage. Below we select the z-axis as the spin quantization axis for $\uparrow$, $\downarrow$. Note that the extrinsic SO coupling generates only the out-of-plane polarized $I_2^{S_z} = -I_3^{S_z}$ component of the transverse spin Hall current, while the Rashba SO coupling leads to both $I_2^{S_z} = -I_3^{S_z}$ and $I_2^{S_x} = -I_3^{S_x}$ as the spin Hall response [11].

The consequences of eq. (3) can be worked out either by analytical means, such as the random matrix theory applicable to "black-box" disordered or chaotic ballistic structures, or by switching to non-perturbative real⊗spin space Green functions [5] to take into account the microscopic Hamiltonian which models details of impurity scattering and possibly strong SO coupling effects. For this purpose, we represent the Rashba [11] and the Thomas term [24] in eq. (1) in the local orbital basis

$$\begin{aligned} \hat{H} \; = & \sum_{\mathbf{m},\sigma} \varepsilon_{\mathbf{m}} \hat{c}_{\mathbf{m}\sigma}^{\dagger} \hat{c}_{\mathbf{m}\sigma} + \sum_{\langle \mathbf{m}\mathbf{m}' \rangle} \sum_{\sigma\sigma'} \hat{c}_{\mathbf{m}\sigma}^{\dagger} t_{\mathbf{m}\mathbf{m}'}^{\sigma\sigma'} \hat{c}_{\mathbf{m}'\sigma'} \\ & - i\lambda_{\rm SO} \sum_{\mathbf{m},\alpha\beta} \sum_{ij} \sum_{\nu\gamma} \epsilon_{ijz} \nu\gamma (\varepsilon_{\mathbf{m}+\gamma\mathbf{e}_j} - \varepsilon_{\mathbf{m}+\nu\mathbf{e}_i}) \\ & \times \hat{c}_{\mathbf{m},\alpha}^{\dagger} \hat{\sigma}_{\alpha\beta}^{z} \hat{c}_{\mathbf{m}+\nu\mathbf{e}_i+\gamma\mathbf{e}_j,\beta}. \end{aligned} \tag{4}$$

The first term models the isotropic short-range spin-independent static impurity potential where $\varepsilon_{\mathbf{m}} \in [-W/2, W/2]$ is a uniform random variable. The second, the Rashba term, is a tight-binding type of Hamiltonian whose nearest-neighbor $\langle \mathbf{m}\mathbf{m}' \rangle$ hopping is a non-trivial 2×2 Hermitian matrix $\mathbf{t}_{\mathbf{m}'\mathbf{m}} = (\mathbf{t}_{\mathbf{m}\mathbf{m}'})^{\dagger}$ in the spin space

$$\mathbf{t}_{\mathbf{m}\mathbf{m}'} = \begin{cases} -t_{\rm o} {\rm I}_s - i t_{\rm SO} \hat{\sigma}_y & (\mathbf{m} = \mathbf{m}' + \mathbf{e}_x), \\ -t_{\rm o} {\rm I}_s + i t_{\rm SO} \hat{\sigma}_x & (\mathbf{m} = \mathbf{m}' + \mathbf{e}_y). \end{cases} \tag{5}$$

Here $\mathbf{I}_s$ is the unit 2×2 matrix in the spin space. The strength of the SO coupling is measured by the parameter $t_{\rm SO} = \alpha/2a$ which determines the spin-splitting of the band structure $\Delta_{\rm SO} = 4a t_{\rm SO} k_F$. A direct correspondence between the continuous effective Hamiltonian eq. (1) and its lattice version eq. (4) is established by selecting the Fermi energy ($E_F = -3.5 t_{\rm o}$) of the injected electrons to be close to the bottom of the band where the tight-binding dispersion reduces to the parabolic one, and by using $t_{\rm o} = \hbar^2/(2m^* a^2)$ for the orbital hopping which

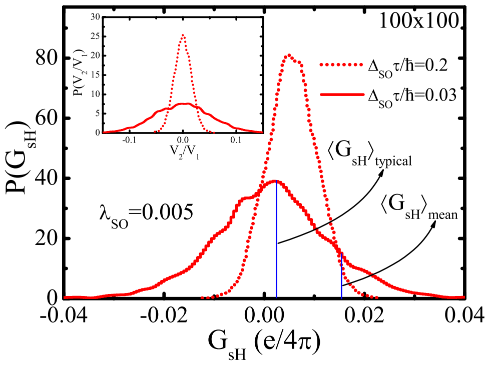

Fig. 2: (Colour on-line) The full distribution function $P(G_{sH})$ of the spin Hall conductance G_{sH} fluctuating from sample to sample in an ensemble of 10000 impurity configurations within 2DEG with the extrinsic SO scattering strength $\lambda_{\rm SO}=0.005$ and the intrinsic Rashba SO coupling $t_{\rm SO}=0.003t_{\rm o}$. The inset shows the corresponding distribution function of voltages V_2 which have to be applied to the top transverse lead (together with voltage V_3 on the bottom lead) to ensure that no charge current $I_2=I_3=0$ accompanies pure spin current $I_2^{S_z}$.

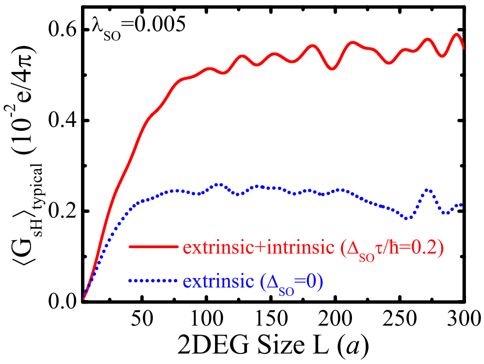

Fig. 3: (Colour on-line) Scaling of the typical spin Hall conductance with the size of the 2DEG (the unit of length is $a\simeq 2\,$nm) characterized by the extrinsic SO scattering strength $\lambda_{\rm SO}=0.005$ and the intrinsic Rashba SO coupling $t_{\rm SO}=0.003t_{\rm o}$ which, together with the disorder setting the mean free path $\ell=v_F\tau=34a$, determine $\Delta_{\rm SO}\tau/\hbar=0.2$. For comparison, the "extrinsic" curve corresponds to $t_{\rm SO}=0=\Delta_{\rm SO}$.

yields the effective mass m^* in the continuum limit. The labels in the third (Thomas) term, which involves second neighbor hoppings, are: the dimensionless extrinsic SO scattering strength $\lambda_{\rm SO}=\lambda\hbar/(4a^2)$; ϵ_{ijz} stands for the Levi-Civita totally antisymmetric tensor with i,j denoting the in-plane coordinate axes; and ν,γ are the dummy indices taking values ± 1. For 2DEG of ref. [3] these SO coupling parameters are set to $t_{\rm SO}\simeq 0.003t_{\rm o}$ and $\lambda_{\rm SO}\simeq 0.005$, assuming conduction bandwidth $\simeq 1\,$eV and using the effective mass $m^*=0.074m_0$.

Mesoscopic fluctuations and scaling of the spin Hall conductance. – Akin to the dichotomy in the description of the quantum Hall effect [22], the characterization of the spin Hall effect in terms of σ_{sH} is (assuming a "reliable" definition of spin current [18]) applicable at sufficiently high temperatures ensuring the classical regime where the dephasing length L_ϕ is much smaller than the sample size and conductivity can be treated as a local quantity. On the other hand, in mesoscopic samples of size $L<L_\phi$ non-local quantum corrections to semiclassical background and the presence of an external measuring circuit have to be taken into account through G_{sH}.

For example, quantum coherence and stochasticity introduced by the impurities will lead to sample-to-sample fluctuations of G_{sH}, so that lack of self-averaging requires to study its full distribution function $P(G_{sH})$, as demonstrated by fig. 2. Nevertheless, the main part of the distribution is best characterized by the typical (*i.e.*, geometric mean) [25] conductance in fig. 2, rather than the usual arithmetic mean value affected by the long tails of $P(G_{sH})$. Therefore, we utilize typical values $\langle G_{sH}\rangle\equiv\langle G_{sH}\rangle_{\rm typical}$ throughout the paper to characterize an ensemble of mesoscopic spin Hall bars differing from each other in the impurity configuration. The inset of fig. 2 also reveals sample-to-sample fluctuations of the voltages [26] that have to be applied to transverse leads to ensure pure spin Hall current $I_2^{S_z}=-I_3^{S_z}\neq 0$ with no net charge current $I_2=I_3=0$ in the transverse direction. Note that in perfectly clean and symmetric bars this condition is trivially satisfied by applying $V_2=V_3=0$ (for chosen voltages $V_1=V/2=-V_4$ in the longitudinal leads) [11].

While the disorder-averaged extrinsic spin Hall conductance $\langle G_{sH}^{\rm extr}\rangle=\langle G_{sH}(\lambda,\alpha\equiv 0)\rangle$ depends linearly on λ, as is the case of the extrinsic bulk spin Hall conductivity $\sigma_{sH}^{\rm extr}\propto\lambda$ [8], fig. 2 demonstrates that in phase-coherent bars mesoscopic fluctuations on the top of semiclassical background affect the change of the sign of G_{sH} from sample to sample and can generate wide distribution $P(G_{sH})$. Nevertheless, we predict both $\langle G_{sH}\rangle_{\rm typical}$ and $\langle G_{sH}\rangle_{\rm mean}$ to be positive within the "dirty" regime corresponding to recent experiments [1,3], meaning that the spin Hall current $I_2^{S_z}$ flows from the top to the bottom transverse lead because of spin-↑ being deflected to the right.

The finite-size scaling of $\langle G_{sH}\rangle(L)$ in fig. 3 reaches to the sample size $L\lesssim 1\,\mu$m, in contrast to $\sim 100\,\mu$m samples required for optical scanning of the spin Hall accumulation [1,3]. Nevertheless, fig. 3 can in principle be used to account for the spin Hall quantities of a microstructure if we view it as a classical network of resistors of the size $L_\phi\times L_\phi$. The conductance $\langle G_{sH}\rangle(L=L_\phi)$ of each resistor can be read from fig. 3, as computed in fully phase-coherent regime with localization length $\xi\gg L_\phi$. Even though inelastic scattering will destroy the quantum-coherent nature of transport between resistors, thereby pushing it into the semiclassical regime [10,14], the spin Hall conductance of the whole network (*i.e.*, of a macroscopic 2DEG bar at high temperatures) is still equal to $\langle G_{sH}\rangle(L=L_\phi)$ in 2D.

Conclusions. – Our principal result in fig. 1 demonstrates that the spin Hall effect in four-terminal disordered 2DEGs can be affected by both the SO scattering of impurities and the Rashba SO coupling inducing the energy spin-splitting $\Delta_{\rm SO}$. The former dominates $G_{sH} \simeq G_{sH}^{\rm extr}$ in the "dirty" regime $\Delta_{\rm SO}\tau/\hbar \lesssim 10^{-1}$, while the latter accounts for $G_{sH} \simeq G_{sH}^{\rm intr}$ in the "clean" regime $\Delta_{\rm SO}\tau/\hbar \gtrsim 1$, where it could be tuned via the gate electrode [27] to two orders of magnitude larger value (for presently achievable Rashba coupling strengths $t_{\rm SO} \sim 0.01$ [27]) than the maximum value of $G_{sH}^{\rm extr}$ reached in the "dirty" limit. In the "crossover" regime $10^{-1} \lesssim \Delta_{\rm SO}\tau/\hbar \lesssim 1$, our non-perturbative quantum transport analysis unveils an interplay of the extrinsic and intrinsic mechanisms responsible for slight deviations of G_{sH} from $G_{sH}^{\rm extr} + G_{sH}^{\rm intr}$. These predictions offer a clear guidance toward nanofabrication of high-mobility devices with stronger structural iversion asymmetry in order to increase the magnitude of the spin Hall current, as well as to evade large sample-to-sample fluctuations of G_{sH} and its sign change in fig. 2 characterizing the low-temperature spin Hall quantum transport in dirty phase-coherent bars.

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We are grateful to B. I. Halperin, N. Nagaosa, and K. Nomura for inspiring discussions. This research was supported in part by the University of Delaware Research Foundation and ACS grant No. PRF-41331-G10.

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