Spin Hall Current Driven by Quantum Interferences in Mesoscopic Rashba Rings

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We propose an all-electrical nanostructure where *pure* spin current is induced in the transverse voltage probes attached to a quantum-coherent ballistic one-dimensional ring when unpolarized charge current is injected through its longitudinal leads. Tuning of the Rashba spin-orbit coupling in a semiconductor heterostructure hosting the ring generates quasiperiodic oscillations of the predicted spin-Hall current due to *spin-sensitive quantum-interference effects* caused by the difference in the Aharonov-Casher phase accumulated by opposite spin states. Its amplitude is comparable to that of the spin-Hall current predicted for finite-size (simply connected) two-dimensional electron gases, while it gets reduced gradually in wide two-dimensional rings or due to spin-independent disorder.

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Introduction.—The increasing interest in spin-based information processing has fomented the field of *semiconductor spintronics* [1,2] where a plethora of concepts, exploiting fundamental quantum phenomena which involve electron spin, have arisen in order to generate and measure *pure spin currents.* In contrast to conventional charge currents, or spin-polarized charge currents that have been explored and utilized in metal spintronics over the past two decades [3], pure spin currents emerge when an equal number of spin-$\uparrow$ and spin-$\downarrow$ electrons move in the opposite direction so that the net charge current is zero [4]. Early [5] and recent [6–10] theoretical analysis has found potential sources of such currents in metallic or semiconductor paramagnets with spin-orbit (SO) dependent scattering on impurities (supporting the *extrinsic* spin-Hall effect [5,6] as transverse spin current in response to longitudinal charge transport, or skew-scattering effects in Y-shaped semiconductor junctions [7]), multiprobe ferromagnet–normal-metal hybrid devices [8], optical injection in clean semiconductors [9], and adiabatic spin pumping in mesoscopic systems [10]. Moreover, spin currents without accompanying charge currents have been generated and detected in optical pump-probe experiments [11] and semiconductor quantum spin pumps [12].

Recent theoretical hints of the existence of the *intrinsic* spin-Hall effect in hole-doped [13] or electron-doped [14] semiconductor systems, whose Bloch energy bands are spin split due to SO couplings [2], have attracted considerable attention. This is essentially a *semiclassical* effect in infinite homogeneous systems where *pure* spin current j_y^z (substantially larger than in the case of the extrinsic effect) is predicted to transport z-polarized spins along the transverse direction (y axis) in response to longitudinal external electric field E_x. The generation and control of a sizable pure spin-Hall current could open new avenues for *all-electrical* spin manipulation without the need for external magnetic fields or problematic coupling of ferromagnetic electrodes to semiconductor devices [2].

The *nonequilibrium* spin current represents the transport of spins between two locations in real space. However, intense theoretical striving to understand the nature of intrinsic spin-Hall current, quantified by j_y^z [15] and the spin-Hall conductivity $\sigma_{\mathrm{sH}} = j_y^z/E_x$, suggests that $j_y^z \neq 0$ might not imply [16,17] real transport of spins in the "dissipationless" regime [13,14] that *lacks* explicit time-reversal symmetry breaking. In addition, perturbative studies concerned with the influence of disorder (i.e., spin-independent scattering off static impurities) on the intrinsic effect [18], as well as reexamination of the original arguments for ballistic systems [19], infer that $\sigma_{\mathrm{sH}} \to 0$ in the *bulk* of a two-dimensional electron gas (2DEG) whose Rashba SO coupling [2] stems from the structure inversion asymmetry of the heterostructure quantum well.

Nevertheless, quantum transport analysis of *measurable* [11,20] spin-resolved charge currents $I_p^\uparrow$ and $I_p^\downarrow$, and corresponding spin currents $I_p^s = \frac{\hbar}{2e}(I_p^\uparrow - I_p^\downarrow)$, in the ideal leads (without SO interaction) of multiprobe Hall bars accessible to experiments predicts that *mesoscopic* spin-Hall current will appear in the transverse voltage probes [20–22] attached to a *finite-size* 2DEG with Rashba SO interaction. This is due to the fact that spin currents in both the diffusive and the ballistic regimes can be facilitated by macroscopic inhomogeneities [19]. Furthermore, intrinsic spin-Hall accumulation [23] on the lateral edges of a two-probe structure has been detected in very recent experiments on spin-split 2D hole gases [24].

However, no experiment detecting the spin-Hall current itself (at least indirectly [4,6]) has been reported yet. Thus, it is intriguing to pose two fundamental questions: Does phase coherence (i.e., spin and orbital quantum-interference effects) play any role in spin-Hall current induction that can leave uniquely quantum and experimentally observable signatures? Is it possible to generate spin-Hall current in strictly one-dimensional systems with no bulk? In this Letter we undertake answering both of these questions by analyzing the spin-charge quantum transport in the presence of the Rashba SO coupling within a mesoscopic ring-shaped conductor (Fig. 1), which is modeled by the following single-particle effective mass Hamiltonian

$$\hat{H} = \frac{\hat{\mathbf{p}}^2}{2m^*} + \frac{\alpha}{\hbar}(\hat{\boldsymbol{\sigma}} \times \hat{\mathbf{p}})_z + V_{\rm conf}(x, y) + V_{\rm dis}(x, y). \quad (1)$$

Here $\hat{\boldsymbol{\sigma}}$ is the vector of the Pauli spin operator, $\hat{\mathbf{p}}$ is the momentum operator in 2D space, α is the strength of the Rashba SO coupling [2], $V_{\rm conf}(x, y)$ is the potential confining electrons to a finite ring region, and static random potential $V_{\rm dis}(x, y)$ accounts for spin-independent impurities. Such a *Rashba ring,* attached to two longitudinal current probes and two transverse voltage probes (Fig. 1), will generate spin-Hall current in the transverse leads. As demonstrated in Fig. 2 for 1D and in Fig. 3 for 2D *clean* $[V_{\rm dis}(x, y) = 0]$ rings, the spin-Hall conductance $G_{\rm sH}^z = I_2^s/(V_1 - V_4)$ measuring the magnitude of *pure* ($I_2 \equiv I_2^\uparrow + I_2^\downarrow = 0$) spin currents in multiprobe mesoscopic structures [20–22] exhibit quasiperiodic oscillations when Rashba SO coupling is increased (e.g., via the gate electrode covering the ring [25]).

The ring conductors smaller than the dephasing length $L_\phi \lesssim 1\ \mu$m (for low temperatures $T \ll 1$ K) have played an essential role in observing how *coherent superpositions* of quantum states (i.e., quantum-interference effects) on the mesoscopic scale leave imprints on measurable transport properties [26]. That is, they represent a solid state realization of a two-slit experiment—an electron entering the ring can propagate in two possible directions (clockwise and counterclockwise) where superpositions of corresponding quantum states are sensitive to the acquired topological phases in a magnetic [Aharonov-Bohm (AB) effect] or an electric [Aharonov-Casher (AC) effect for particles with spin] external field whose changing generates an oscillatory pattern of the ring conductance [26,27]. Moreover, the recently proposed all-electrical spintronic 1D ring device [28] would utilize *spin interferences* [29–31] (i.e., the AC phase difference acquired by opposite spin states during their cyclic evolution around the ring) to modulate the conductance of conventional unpolarized current (injected through single-channel leads) between 0 and $2e^2/h$ by changing the Rashba coupling [28,29,31].

Quantum transport of spin currents in 4-probe rings.—The charge currents I_p in a mesoscopic structure attached to many leads (labeled by p) are described by the multiprobe Landauer-Büttiker formulas [32]

$$I_p \equiv I_p^\uparrow + I_p^\downarrow = \sum_q G_{pq}(V_p - V_q), \quad (2)$$

while the analogous formulas for spin currents in the leads are straightforwardly extracted from them [7,20]

$$I_p^s \equiv \frac{\hbar}{2e}(I_p^\uparrow - I_p^\downarrow) = \frac{\hbar}{2e}\sum_q (G_{qp}^{\rm out} V_p - G_{pq}^{\rm in} V_q). \quad (3)$$

Here $G_{pq}^{\rm in} = G_{pq}^{\uparrow\uparrow} + G_{pq}^{\uparrow\downarrow} - G_{pq}^{\downarrow\uparrow} - G_{pq}^{\downarrow\downarrow}$ and $G_{qp}^{\rm out} = G_{qp}^{\uparrow\uparrow} + G_{qp}^{\downarrow\uparrow} - G_{qp}^{\uparrow\downarrow} - G_{qp}^{\downarrow\downarrow}$ have a transparent physical interpretation: $\frac{\hbar}{2e} G_{qp}^{\rm out} V_p$ is the spin current flowing from the lead p with voltage V_p into other leads $q \neq p$ whose voltages are V_q, while $\frac{\hbar}{2e} G_{pq}^{\rm in} V_q$ is the spin current flowing from the leads $q \neq p$ into the lead p. The standard charge conductance coefficients are expressed in terms of the spin-resolved conductances as $G_{pq} = G_{pq}^{\uparrow\uparrow} + G_{pq}^{\uparrow\downarrow} + G_{pq}^{\downarrow\uparrow} + G_{pq}^{\downarrow\downarrow}$ [33]. The linear response conductance coefficients are related to the transmission matrices $\mathbf{t}^{pq}$ between the leads p and q through the Landauer-type formula $G_{pq}^{\sigma\sigma'} = \frac{e^2}{h}\sum_{i,j=1}^{M_{\rm leads}} |\mathbf{t}_{ij,\sigma\sigma'}^{pq}|^2$, where $|\mathbf{t}_{ij,\sigma\sigma'}^{pq}|^2$ is the probability for spin-σ' electron incident in the conducting channel j

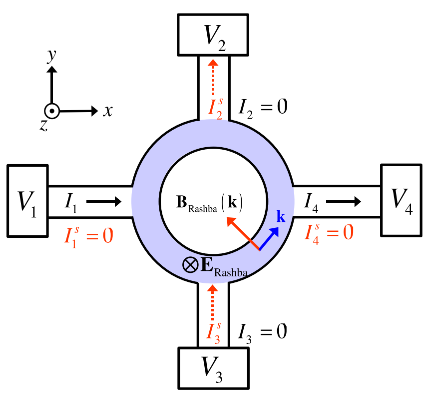

FIG. 1 (color online). The mesoscopic circuit predicted to generate a *pure* z-axis polarized spin-Hall current $I_2^s = \frac{\hbar}{2e}(I_2^\uparrow - I_2^\downarrow) = -I_3^s$ in the transverse voltage probes ($V_2 = V_3 \neq 0$, $I_2 = I_3 = 0$) attached to a ring realized using 2DEG in a semiconductor heterostructure [27]. The injected unpolarized ($I_1^s = 0$) current through (single-channel) longitudinal leads is subjected to the Rashba SO interaction (nonzero in the shaded ring region), which acts as the momentum-dependent effective magnetic field $\mathbf{B}_{\rm Rashba}(\mathbf{k})$ arising due to the electric field $\mathbf{E}_{\rm Rashba}$ responsible for the electron confinement to 2DEG.

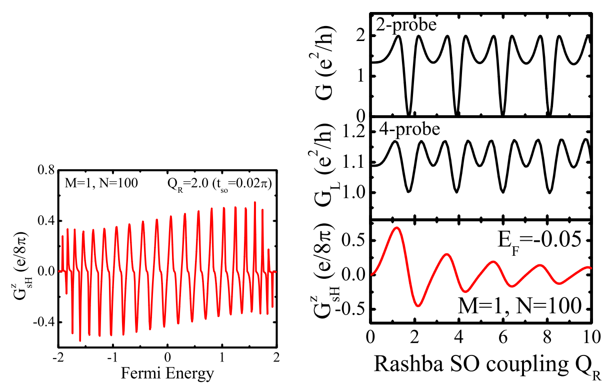

FIG. 2 (color online). The spin-Hall conductance $G_{\rm sH}^z$ (corresponding to the detection of the z component of pure spin current I_2^s) for the 1D ring ($M = 1$, $N = 100$ lattice sites around the ring) attached to four single-channel leads as a function of the Fermi energy $E_{\rm F}$ and the dimensionless Rashba SO coupling $Q_{\rm R} \equiv (\alpha/2at_0)N/\pi$. The right panel also plots the longitudinal charge conductance $G_L(Q_{\rm R}) = I_4/(V_1 - V_4)$ of our four-terminal ring depicted in Fig. 1, as well as the charge conductance $G(Q_{\rm R})$ of the corresponding two-terminal AC ring [29,31] where the transverse leads are removed.

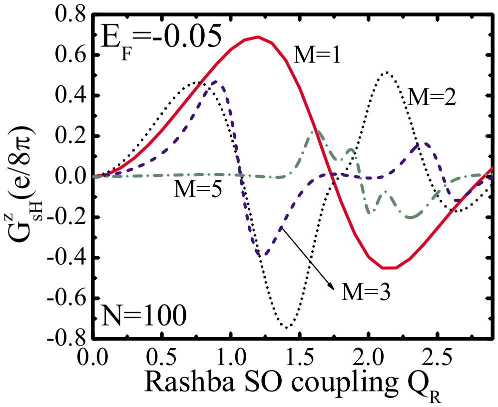

FIG. 3 (color online). The modulation of the spin-Hall conductance G^z_{sH} by changing the Rashba SO coupling $Q_R \equiv (\alpha/2at_0)N/\pi$ in 2D ballistic rings of finite width (modeled by $M \geq 1$ coupled concentric 1D ring chains of $N = 100$ lattice sites) attached to four single-channel leads.

of lead q to be transmitted to the conducting channel i in the lead p as the spin-σ electron [32]. The most general expression for the spin-Hall conductance of a 4-probe structure is given by [20]

$$G_{sH} = \frac{\hbar}{2e}\left[(G^{out}_{12} + G^{out}_{32} + G^{out}_{42})\frac{V_2}{V_1} - G^{in}_{23}\frac{V_3}{V_1} - G^{in}_{21}\right], \tag{4}$$

where we choose the reference potential $V_4 = 0$. Although all three components of the polarization vector of transported spin in the lead 2 are nonzero, only the z component of the spin current exhibits the property $(I^s_2)_z = -(I^s_3)_z$ of the spin-Hall effect [20]. Therefore, we focus on G^z_{sH} by setting the z axis as the spin quantization axis for ↑, ↓ in Eq. (4).

We recall that the Landauer transport paradigm spatially separates single-particle coherent and many-body inelastic processes by attaching the sample to huge electron reservoirs where, in order to simplify the scattering boundary conditions, semi-infinite ideal leads with vanishing spin and charge interactions are inserted between the reservoirs and the scattering region [32]. Thus, even in the ballistic regime dissipation effects establishing steady state transport are always incorporated (with point contact conductance playing the role of a time-reversal symmetry breaking parameter), in contrast to artifacts of the Kubo formalism which maps an infinite homogeneous system with an electric field driven spin-Hall current in the dissipationless regime [13,14] (with no impurities) to an equivalent system containing only equilibrium spin currents [16]. Here we also clarify that apparent equilibrium solutions of the multiprobe spin current formulas Eq. (3), $V_q = \text{const} \Rightarrow I^s_p \neq 0$, found in Ref. [7], actually *do not exist.* When all leads are at the same potential, a purely equilibrium term $\frac{\hbar}{2e}(G^{out}_{pp}V_p - G^{in}_{pp}V_p)$ (omitted in Ref. [7]) becomes relevant for I^s_p, canceling all other terms in Eq. (3) to ensure that no *unphysical* spin currents $I^s_p \neq 0$ exist in the leads of an unbiased ($V_q = \text{const}$) mesoscopic structure.

We employ the real ⊗ spin space Green function technique [20,33] to get the *exact* (within single-particle picture) transmission matrix $\mathbf{t}^{pq}$ between the leads p and q. The nonperturbative retarded Green function can be computed efficiently in a local orbital basis representation of the Hamiltonian Eq. (1), as introduced in Ref. [29] as a set of M concentric ring chains composed of N lattice sites spaced at a distance a. The characteristic energy scales of such a lattice Hamiltonian are the hopping between neighboring sites $t_0 = \hbar^2/(2m^*a^2)$ [all energies are measured in the units of t_0] and the Rashba hopping $t_{so} = \alpha/2a$. The Rashba SO coupling within the ring region is quantified by a dimensionless parameter $Q_R \equiv (t_{so}/t_0)N/\pi$ [29,31]. Using a quantum point contact at the ring-lead interface can ensure that the unpolarized current is injected through a *single* open conducting channel. Therefore, we assume 1D electrodes ($M_{leads} = 1$) while allowing for both strictly 1D rings $M = 1$ and 2D rings of finite width $M > 1$ [29].

Spin-interference effects in spin-Hall conductance.— The rapid oscillations of $G^z_{sH}(E_F)$ in Fig. 2 arise due to the discrete nature of the energy spectrum in an isolated ring [30] (the attached leads renormalize these eigenlevels and inflict their *finite width* since the electron spends finite time inside the ring before escaping toward the reservoirs). The charge conductance of the two-probe 1D AC ring [28,29,31] becomes zero at specific values of Q^{min}_R for which *destructive* spin interference of opposite spins traveling in opposite directions around the ring takes place. For example, in a simplified treatment [31] $G = \frac{e^2}{h} \times (1 - \cos\frac{\Phi^{\uparrow}_{AC} - \Phi^{\downarrow}_{AC}}{2})$, where $\Phi^{\sigma}_{AC} = \pi(1 + \sigma\sqrt{Q^2_R + 1})$ is the AC phase acquired by a spin-↑ or a spin-↓ quantum state ($\sigma = \pm$ for ↑, ↓), has minima $G(Q^{min}_R) = 0$ at $Q^{min}_R \simeq \sqrt{n^2 - 1}$ ($n = 2, 4, 6, \ldots$). However, adding two transverse leads onto the same 1D ring lifts the minima of the longitudinal conductance to $G_L(Q^{min}_R) = I_4/(V_1 - V_4) \simeq e^2/h$ (Fig. 2) due to the contribution from incoherent (indirect) paths, 1 → 2 → 4 and 1 → 3 → 4, which do not exhibit destructive spin-interference effects that characterize coherent (direct) paths from terminals 1 to 4. Nevertheless, G^z_{sH} vanishes at Q^{min}_R, while the amplitude of its quasiperiodic oscillations (which are absent in simply connected mesoscopic spin-Hall bridges [20]) gradually decreases at large Q_R because of the reflection at the ring-lead interface [33].

Finally, we examine the observability of the quantum spin-Hall current in realistic rings of finite width and in the presence of impurities. Figure 3 demonstrates that in ballistic 2D rings attached to four single-channel probes the fingerprints of the quantum-interference dominated spin-Hall effect—$G^z_{sH}(Q^{min}_R) = 0$ at specifically tuned (but harder to interpret [29,30]) Q^{min}_R—can survive. When $M = 2$, we observe that the frequency of $G^z_{sH}(Q_R)$ oscillations is

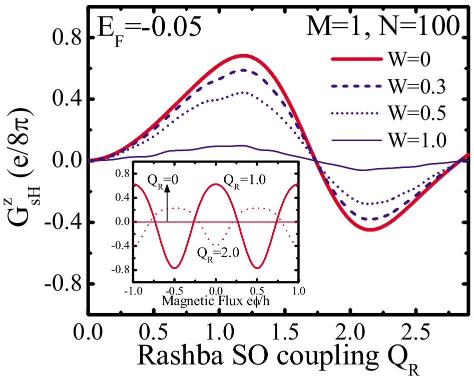

FIG. 4 (color online). The effect of the disorder ($\ell_{W=0} \rightarrow \infty$, $\ell_{W=0.3} \approx 5.3L_c$, $\ell_{W=0.5} \approx 1.9L_c$, $\ell_{W=1.0} \approx 0.5L_c$, where $L_c = r_0\pi/2$) on the spin-Hall conductance $G^z_{\rm sH}$ of the 1D ring whose ballistic regime is examined in Fig. 2. The inset shows AB oscillations of $G^z_{\rm sH}(\phi)$ when the ballistic ring ($W = 0$) is penetrated by an external magnetic flux $\phi = B_z \pi r_0^2$.

almost doubled. This is due to the presence of the second harmonic in the ring, which is a well-known effect in the AB rings with a large radius/width ratio [34]. At larger widths, the quasiperiodicity of the $G^z_{\rm sH}(Q_{\rm R})$ is destroyed since accumulated AC phases average over intertwined 1D closed paths through the ring [35], thereby "dephasing" the *visibility* of spin-interference effects [29]. The salient features of $G^z_{\rm sH}$ are resilient to weak disorder, modeled as the on-site random potential $\varepsilon_{\mathbf{m}} \in [-W/2, W/2]$ setting a finite mean free path $\ell = 3(4t_0^2 - E_{\rm F}^2)a/W^2$ (in the Born approximation), which is able to reduce only the amplitude of the 1D ring transport properties [35].

Conclusions.—The predicted "quantum" spin-Hall effect is indirectly observable via measuring its *unequivocal* experimental signature—a quasioscillatory pattern (as a function of the tunable Rashba coupling [25]) of the voltage induced [4,6,22] by the flow of the pure spin current exiting from the Rashba spin-split *multiply connected* region through single-open-channel electrodes. To observe these oscillations at finite temperatures, the width of the distribution of injected electrons should not exceed the gap between the adjacent peaks of $G^z_{\rm sH}(E_{\rm F})$ in Fig. 2, while its center (i.e., $E_{\rm F}$ of the reservoirs) should be adjusted to their position. At fixed SO coupling, the AB-type oscillations [26,27] of the spin-Hall current and associated voltages can also be utilized by introducing magnetic flux through the ring, as suggested by the inset of Fig. 4.

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